PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
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The Colorimetric Properties of the Spectrum

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V. *The Colorimetric Properties of the Spectrum.*

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(*Communicated by Sir* JOSEPH PETAVEL, *F.R.S.*)

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Introduction.

Those properties of the eye which determine its behaviour in the measurement of luminous intensity and colour are completely defined by two sets of numerical data which express, as functions of wave-length, its behaviour to monochromatic radiation throughout the visible spectrum in respect of these two aspects of the visual effect of a stimulus. The first of these functions is embodied in the "Relative Visibility"* curve of the spectrum, and the second is embodied in a curve showing the locus of the spectrum on the "colour triangle" of some trichromatic system.

These two functions may be combined to give the "mixture curves" of the spectrum, by means of which we can calculate both the photometric and colorimetric values of any stimulus from its spectral energy distribution.

The nature and significance of these various functions are sufficiently well understood to need no explanation here.

The visibility function of the normal human eye has been known to a high degree of accuracy for some time, and in 1924 the International Commission of Illumination agreed to accept, as representing the normal eye for photometric purposes, a set of data based on an analysis of all the modern determinations.† Except for some uncertainty at the two extremities of the spectrum these data are unquestionably closely representative of the normal eye.

No such accumulation of data exists for the colorimetric properties of the eye. CLERK MAXWELL‡ first determined the spectrum locus on a colour triangle constructed in accordance with NEWTON's law of colour mixture, and also deduced mixture curves of the spectrum for himself and his assistant, using as primaries spectral radiations of wave-lengths 0·6307 μ, 0·5286 μ and 0·4573 μ, approximately.

* The writer is not alone in objecting to the use of the word "visibility" in connection with the luminosity function, but prefers to continue its use under protest until some more acceptable term is substituted by general agreement.

† 'Recuil des Travaux' p. 67 (1924); see also K. S. GIBSON and E. P. TYNDALL, 'Bur. Stds. Sci. Papers,' No. 475 (1923).

‡ 'Sci. papers,' vol. 1, pp. 410–444; 'Phil. Trans.,' vol. 150, p. 67 (1860).

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MAXWELL's results indicate that the spectrum he employed was very impure, particularly in the blue-green part of the spectrum. This gave an entirely fictitious approximation of the spectrum locus to two sides of the colour triangle and led MAXWELL to conclude that ". . . all the colours of the spectrum may be compounded of those which lie at the angles of this triangle," and that there was "strong reason to believe that these are the three primary colours corresponding to those modes of sensation in the organ of vision."

These conclusions have long been known to be quite untenable, there being no spectral colours which are in any sense *Primary* colours or from which all other colours can be compounded.

Determinations by other methods were made by KÖNIG and DIETERICI* and by ABNEY.†

Certain modifications in KÖNIG's data were made by IVES‡ to obtain better agreement with luminosity relationships; and WEAVER§ analysed the results of KÖNIG and DIETERICI and ABNEY, reduced them to a common denominator, and obtained a set of values based on them which was recommended for general use by the Colorimetry Committee of the Optical Society of America in 1922; but, as far as the writer is aware, no attempt has been made, since those of KÖNIG and ABNEY, to obtain further *experimental* information on the chromatic functions until quite recently.

The tendency to elaborate and correct the meagre data of the earlier investigations, rather than to initiate fresh experimental work on the subject under modern experimental conditions, has no doubt been due to difficulties, supposed to be inherent in such investigations, which have only been removed by the progress made on the theoretical side of the subject during the last ten or fifteen years. IVES‡ || first directed attention to the simple relations existing between the different modes of presentation of the facts of colour mixture, and in particular pointed out that if the trichromatic coefficients of the spectrum (coordinates of the spectrum locus on the colour triangle) were known for any one set of primaries they were, in fact, known for any other possible set; and similarly for the spectral mixture curves.

Following on the work of IVES, the present writer¶ simplified some of the transformations involved and, in particular, showed that the trichromatic coefficients of the spectrum could be measured on any trichromatic colorimeter, and the results converted to any desired set of standard spectral primaries by a transformation requiring no additional experimental work.

Towards the end of 1926, in the course of the work on colorimetric standardisation

* 'Z. Psychol. Physiol,' vol. 4, p. 241 (1892).
† 'Phil. Trans.,' A, vol. 205, p. 333 (1905).
‡ 'J. Frank. Inst.,' vol. 180, p. 673 (1915).
§ 'J. Amer. Opt. Soc.,' vol. 6, p. 527 (1922).
|| 'J. Frank. Inst.,' vol. 195, p. 23 (1923).
¶ 'Trans. Opt. Soc.,' vol. 26, p. 95 (1924–25).

carried out at the National Physical Laboratory, it became necessary to obtain more reliable values for the spectrum than those available. It was only possible at the time to carry out the measurements for seven observers, but this was regarded as sufficient for the internal purposes for which the results were immediately wanted. The work was carried out, and a colour triangle and set of mixture curves were prepared, as presently to be described.

These results were not published for general use because it was at first thought that seven observers constituted much too small a group to give a reasonably close approximation to a "normal" eye, and it was intended to extend the measurements to a very much larger group at a later date. Pressure of other work prevented this intention from being carried out as soon as had been hoped. Meanwhile the seven-observer data had been used for various purposes, and had proved surprisingly satisfactory.

During this period a similar investigation was carried out by W. D. WRIGHT at the Imperial College of Science. WRIGHT used an entirely different type of apparatus* from that employed at the National Physical Laboratory, but adopted the standard conditions recommended by the Laboratory as regards field size and "white" light. He obtained the trichromatic coefficients of the spectrum for ten observers, and published these† in terms of three spectral primaries of wave-lengths, 0·65 μ (red), 0·53 μ (green), and 0·46 μ (blue). In a later paper‡ WRIGHT has calculated mixture curves of the spectrum based on his own determination of the trichromatic coefficients and the International Visibility curve.

The trichromatic coefficients for his ten observers agreed so closely with those of the seven observers examined at the National Physical Laboratory as to indicate that both groups must give results approximating more closely to "normal" than might have been expected from the size of either group, and made it desirable to publish the N.P.L. results without waiting for measurements on additional observers. It was hoped that international agreement on a standard white light for colorimetry might have been reached during 1930, and that the results presented in this paper could be given on that basis; but such agreement has not yet been attained and it is felt undesirable to withhold publication any longer. If international discussion should result in the general adoption of a standard white differing from that hitherto adopted at the N.P.L., the values here given can easily be converted to such new basis.

Experimental Arrangement.

The colorimeter employed for the determination of the trichromatic coefficients was made by Adam Hilger, Ltd., and is of the type designed by the writer for general

* 'Trans. Opt. Soc.,' vol. 29, p. 225 (1927–28).

† 'Trans. Opt. Soc.,' vol. 30, p. 141 (1928–29).

‡ 'Trans. Opt. Soc.,' vol. 31, p. 201 (1929–30).

colorimetric purposes. As it has been fully described elsewhere,* it is merely necessary to say here that the working primaries of the instrument are obtained by passing the light from an opal-bulb gas-filled lamp through red, green and blue gelatine filters. The three primary stimuli so obtained can be blended in any desired proportions in the matching field of the instrument. The particular filters employed were chosen so that the great majority of colours met with in industry and commerce can be matched by some blend of the three primaries, but provision is made for transferring any one of the primaries to the same side of the matching field as the colour under test in order to deal with colours which lie outside the colour triangle of the instrument primaries. The transferred primary appears with a negative coefficient in the result. As the spectrum locus lies completely outside the colour triangle for any set of real primaries, this procedure of adding one of the primaries to the colour under test and matching the combination by means of the others is necessary in all measurements on the spectrum.

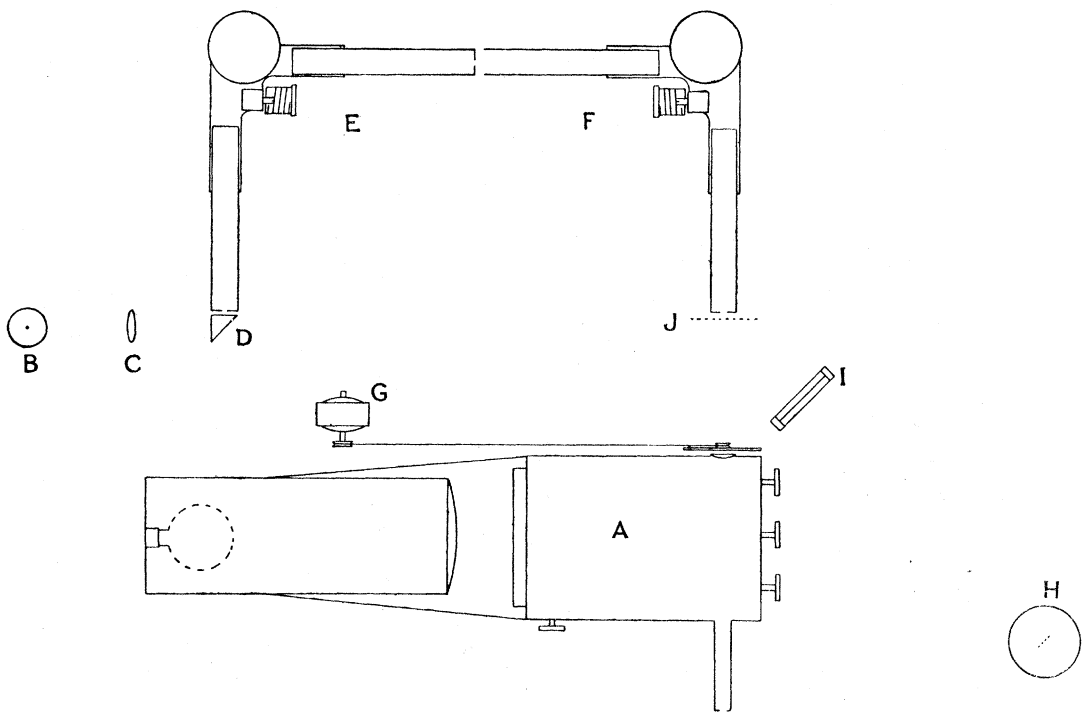

FIG. 1.

The arrangement of apparatus is shown in diagrammatic plan in fig. 1. The colorimeter, A, has a square field of view, divided into two rectangular portions by a horizontal dividing line. One of these portions, the test field, contains the colour to be matched. The other, the matching field, contains a blend of the instrument primaries, which are regulated by the three control handles on the right, their amounts being measured on accurately divided scales attached to the shutters which determine the amount of light passing through each of the filters.

* Guild, 'Trans. Opt. Soc.,' vol. 27, p. 106 (1925–26).

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When it is necessary to introduce any one of the primaries into the test field, as in the present work, the amount so introduced is regulated by the control handle at the front of the instrument. No separate scale is provided for evaluating the amount of primary so transferred. It is evaluated on the main scale of that primary, as explained later.

The size of the aperture which bounds the field of view is such that each side of the square subtends an angle of approximately 2° at the observer's eye. This size of field was chosen for the following reasons :—

It lies almost entirely within the average "yellow spot" of the retina, and is of similar dimensions to the field to which the standard visibility data apply. It is of the greatest importance that quantitative work on the properties of the eye should all apply to the same region of the retina, and colour-matching apparatus with fields extending beyond the macula lutea may give results which are not at all representative of foveal vision. Experiments also showed that for such small fields the simple two-part division was the most advantageous. The Lummer-Brodhun contrast patches, which provide a great increase in sensitivity, either in photometry or colour matching, could not readily be incorporated in such a small field. The use of a tripartite field, which is sometimes advocated as a means of reducing errors due to fatigue and successive contrast, was found to involve a definite loss in sensitivity owing to the narrowness of the strips.

The monochromatic light was obtained from a 40-watt automobile headlight lamp, B, the filament of which was focussed by means of an achromatic, spherically corrected lens, C, and prism, D, on the slit of a constant deviation monochromator, E. The exit slit of this instrument serves also as the entrance slit of a second monochromator, F, of which the exit slit is situated at the focus of the lens system, to be described presently, which constitutes the entrance window of the colorimeter.

The exit slit of the second monochromator is made slightly wider than the image, formed at it, of the exit slit of E, so that no part of this image is obscured.

For the normal use of the colorimeter in measuring the colour of materials, the window of the instrument consists of a lens of clear colourless glass, having a focal length of about 20 cm. The specimen is placed at the focus of this lens, and the light passes as a collimated beam through the matching prism, after which a second lens, at the inner end of the observation tube, produces an image of the specimen in the plane of the observation pupil. The purpose of this is to prevent any structure possessed by the specimen, for example, the texture of fabrics, from appearing in the field of view and destroying the sensitivity of the match. For measurements on the spectrum, if the slit of the monochromator is situated at the focus of the lens-window, an image of the spectral line is formed at the observation pupil of the colorimeter and the test field appears filled with light of the appropriate colour. Owing, however, to the fact that the spectral light is concentrated within the limits of a narrow slit at the observer's eye, the appearance of the field is marred by streaks and other defects, due partly to striæ, dust particles, etc., in the optical system, and to the "muscæ volitantes" produced by non-homogeneity in the media of the eye itself. Such a system is, in fact, of the type which one would arrange for the purpose of showing up such defects. Further, while the spectral light is confined to this slit-aperture, the instrument primaries are not, but completely fill the circular aperture of the observation pupil of the colorimeter. The fact that the spectral light is, in effect, seen through a different observation pupil from the other constituents of the colour match has the result that parallax effects are introduced by the slightest movement of the eye. These two causes are sufficient to destroy all precision of measurement. To obtain good matching conditions, in which both fields can be made to look identical, with a sharp, almost invisible, dividing line, it is necessary to ensure not only that the light is uniformly distributed over the cross-section of the beam as it passes the field aperture, but also that it is at least approximately uniformly distributed over the observation pupil through which it enters the eye.

To effect this in the present investigation use was made of a spreading device which the writer has previously described in connection with another type of instrument.* This consists of a crown of rotating lenses, of about 1 metre focal length, as illustrated in fig. 2, situated close to the fixed lens constituting the window of the colorimeter. The axis of rotation of the crown was vertically above the lens-window so that each lens of the crown was co-axial with the fixed lens when passing through the lowest point of its circular orbit. The slit of the second monochromator was placed at the focus of the combined system, so that its image was in the plane of the observation pupil. When the crown of weak lenses is set in rotation, by means of the motor G, fig. 1, each lens, as it passes across the window, causes the image of the slit to move in an arc-shaped path across the observation pupil, and the effect, when the rotation is rapid, is to spread out the image of the slit into a band about six or seven millimetres broad. The brightness of this band fades off towards either side, but it is sufficiently uniform over the central region occupied by the observation pupil, which was kept at a constant diameter of 3 mm. throughout the experiments, to give a field free from the defects described above.

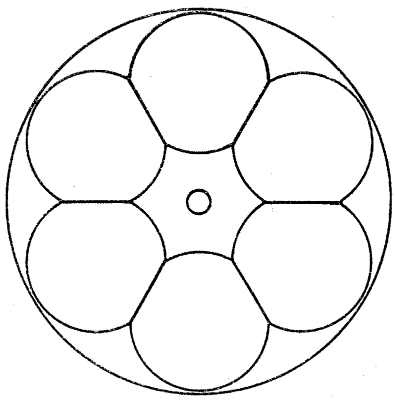

FIG. 2.

In addition to the measurements on the spectrum itself it is necessary to make measurements on a standard white light in order to determine the units of magnitude in which the quantities of the instrument primaries are to be evaluated, in accordance with the usual convention that quantities of the primaries which match white shall

* GUILD, 'Trans. Opt. Soc.,' vol. 27, p. 139 (1925–26).

be measured as equal. The quantity scales of the instrument, on which the actual readings are taken, are, of course, peculiar to the particular instrument employed and have no significance in the final result but, from the measurements on white, factors are found for each of the scales which reduce the numerical results to a system of units independent of the properties of the instrument. Particulars of the standard white light here used are given in Table I* (p. 179). The light was obtained by means of a gas-filled lamp, H, and a liquid filter, I, and was introduced into the field of the colorimeter by inserting a white diffusing surface, J, just in front of the monochromator slit. This surface consisted of magnesium oxide smoked on to a metal plate. The plate was mounted on a swivel joint so that it could be readily swung out of the way to allow passage of the monochromatic light or swung into position for the measurements on white.

An adequate system of screening, not shown in the diagram, was arranged to ensure that no stray light could enter the colorimeter. This was particularly important when measurements near the extremities of the spectrum were in progress. Relatively minute quantities of light, reflected from the jaws of the monochromator slit, or reflected or refracted into the colorimeter by the rotating lenses, would seriously impair the purity of the spectral radiation towards the blue end of the spectrum. It was not found to be either comfortable or convenient to carry out the work with the room in complete darkness, so that precautions had to be taken against stray light. The screening was tested during the preliminary work, and arranged so that no difference could be detected in the measurement of a violet field whether the room was completely dark or illuminated to a convenient extent.

Preliminary tests were also made of the adequacy of the spectral purification effected by the two monochromators by comparing measurements made on blue and violet parts of the spectrum with and without the addition of purifying filters. The results of such tests showed that the purification was sensibly complete.

The wave-length scale of the main monochromator, E, fig. 1, was carefully calibrated against a series of emission spectra. It was refocussed† for each wave-length in accordance with a curve of focus settings obtained by preliminary experiments. The type of focus adjustment provided on the instrument was tested and found to introduce no irregularity in the wave-length calibration.

The slit widths were made as narrow as was consistent with adequate working illumination near the extremities of the spectrum, and were kept at the same widths throughout the work, reduction of illumination in the brighter portions of the spectrum being effected by a resistance in the circuit of the lamp, B.

The precise wave-length range transmitted by the monochromator at different parts of the spectrum was not determined, but preliminary tests were made, with slits of

* See also footnote to p. 163.

† This is accomplished, in the type of instrument employed, by movement of the object glasses, the slits remaining in fixed positions.

approximately double the width, which showed that no appreciable slit-width correction had to be made at any part of the spectrum.

The second monochromator did not require calibration. It had merely to be set so that the image of the exit slit of E was approximately in the centre of the exit slit of F, which, as has already been stated, was appreciably wider than this image.

Experimental Procedure.

When the preliminary work involved in calibrating the various parts of the apparatus had been carried out, the procedure adopted in testing the seven subjects* was as follows. A supervisor, who was thoroughly familiar with the apparatus and the routine to be followed, adjusted the monochromators to the desired wave-length, regulating the intensity of the monochromatic light to a suitable value by means of the resistance in the lamp circuit. He also performed the operation of inserting in the test field a suitable amount of whichever primary had to be "transferred," regulating the amount of this to be a little in excess of that required, so that positive amounts of all three primaries were required to match the resultant colour. It is better to do this than to attempt to transfer the exact amount required to enable the resultant colour to be matched by the two remaining primaries, particularly when testing subjects who are inexperienced in manipulating the instrument. The transferred primary remains constant in amount, producing, in combination with the monochromatic light, a colour which the subject can match by manipulating the three primary controls. It is necessary, however, that the transferred primary should not be very much in excess of requirements, otherwise some sensitivity is lost, particularly near the short wave-length end of the spectrum. Conditions having been adjusted by the supervisor to suit himself, a few preliminary settings were made by the subject to render him familiar with the particular colour match. From these preliminary settings the supervisor judged whether the amount of transferred primary was sufficient for this subject or whether it could profitably be reduced. When optimum conditions had been arrived at a series of ten settings was made, the scale readings being taken down by the supervisor. All three primaries were disturbed between each observation. Usually one such series was sufficient, but if the individual observations varied too much a further set would be taken.

At the conclusion of the series the amount of transferred primary was evaluated. This was done by cutting off the monochromatic light, leaving only the transferred

* The author desires to record his thanks to Dr. A. F. A. Young, Messrs. A. G. Williams, W. J. Stockwell, L. C. Cordle and H. J. Auber, of the staff of the Optics Division of the National Physical Laboratory, and to Miss Lambert, of the staff of the Research Laboratories of the British Woollen and Worsted Industries, who acted as subjects; and to record his further indebtedness to Dr. Young, who conducted the tests on some of the subjects, and performed frequent checks on the lamp and filters used.

primary in the test field, when it could be matched by a suitable setting of the same primary in the matching field, the other two primaries being closed down. The transferred primary was thus evaluated on the same scale as that in the matching field. As such a measurement involves no colour difference, it is unaffected by the peculiarities of the observer's colour vision. It was therefore performed by the supervisor in order to minimise the amount of observational work called for from the subject.

While this measurement was being carried out, and during the readjustment of the apparatus for another wave-length, the subject rested his eye by closing it, or by looking around the moderately lit laboratory, as he felt disposed. Each subject made measurements at wave-length intervals of 0·01 μ from 0·40 μ to 0·48 μ; at intervals of 0·005 μ from 0·48 μ to 0·53 μ; and again at intervals of 0·01 μ from 0·53 μ to 0·70 μ.

Four or five measurements on the standard white, each consisting of ten observations, were interspersed at more or less regular intervals during the spectrum observations. The mean of all the white light measurements made by the subject was used to deduce his "white-light factors" by which his readings on the three primary scales had to be multiplied in order to reduce his measurements to the conventional basis that equal quantities of the three primaries match white.

From his reduced readings, the "unit equations" for the spectrum were obtained, that is, equations of the form

$$Q_\lambda = a\mathrm{R} + b\mathrm{G} + c\mathrm{B}$$

where $a + b + c = 1$. The symbol Q_λ represents the "colorimetric quality" of a monochromatic stimulus of wave-length λ in terms of the three primary stimuli which constitute the working primaries of the colorimeter.

Quantitatively, the amount of the monochromatic stimulus represented by the above equation is one "trichromatic unit," or T. unit for short. A quantity n times as great, or n T. units, is represented by an equation in which the coefficients of the primaries are in the same proportion as a, b and c, but add up to n. The conception of trichromatic units for evaluating stimulus quantities is of very great convenience, as the writer has shown elsewhere,* in facilitating the solution of colour mixture problems.

Results.

The most direct method of exhibiting the results of colorimetric measurements on the spectrum is to plot three separate curves showing the variation with wave-length of the trichromatic coefficients a, b and c of the unit equation. Fig. 3 gives these curves for the seven observers, and illustrates the nature and magnitude of the differences

* 'Trans. Opt. Soc.,' vol. 26, p. 139 (1924–25).

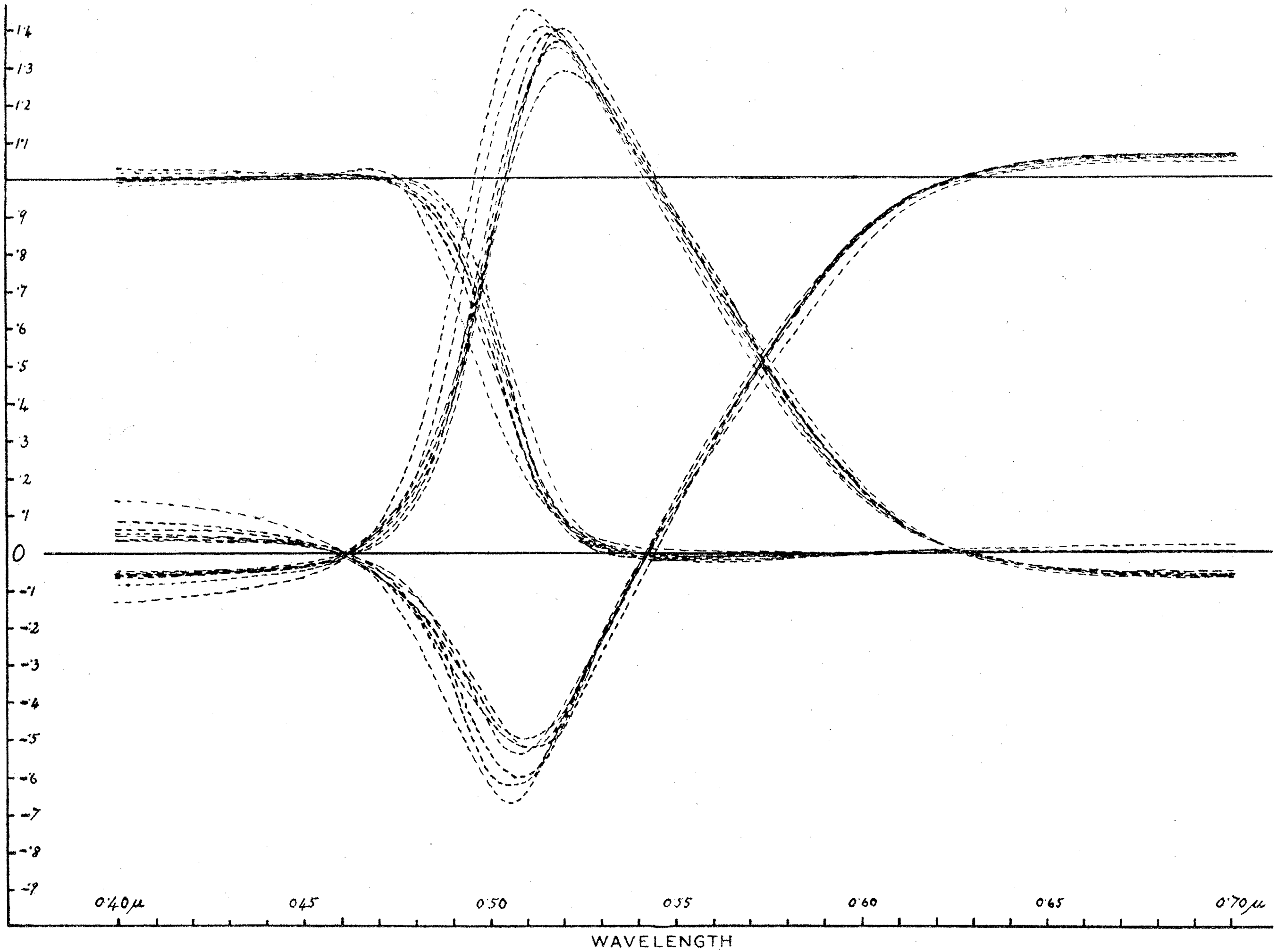

Fig. 3.

exhibited by the members of this group. Fig. 4 gives the mean curves for the group, and in Table II the mean coefficients are given at wave-length intervals of 0·05 μ, the values referring, of course, to the arbitrary and unspecified primaries of the Laboratory colorimeter. As these tabulated coefficients are the basis of all the subsequent calculations, they are given to a greater number of significant figures than have any colorimetric significance in order to prevent accumulation of arithmetical errors in such transformations as may be applied to them. When the mean results were plotted as in fig. 4, the points all lay on perfectly smooth curves. In order, however, to obtain values of a higher order of *numerical* consistency for the small coefficients, these portions of the curves were plotted on a large scale, smoothed out, and the values read from the smoothed curves. This process is adequate for the small coefficients, but, in parts of the spectrum where the curves are steep, ambiguity may exist as to values at wave-lengths intermediate between the experimental points. This uncertainty can be eliminated by a method of interpolation which I have previously used (*loc. cit.*, 1924–25) in obtaining intermediate values from KÖNIG's data. The coefficients were read from the smoothed curves at wave-length intervals of 0·001 μ, and plotted on a large-scale colour triangle. The best locus was drawn through them, care being taken to make it pass through all the experimentally determined points. Errors in the interpolated values were then revealed by points lying appreciably off the plot, and by irregularities of spacing along it. The lateral errors were first corrected, and then measurements were made of the distances of the various points along the locus, as

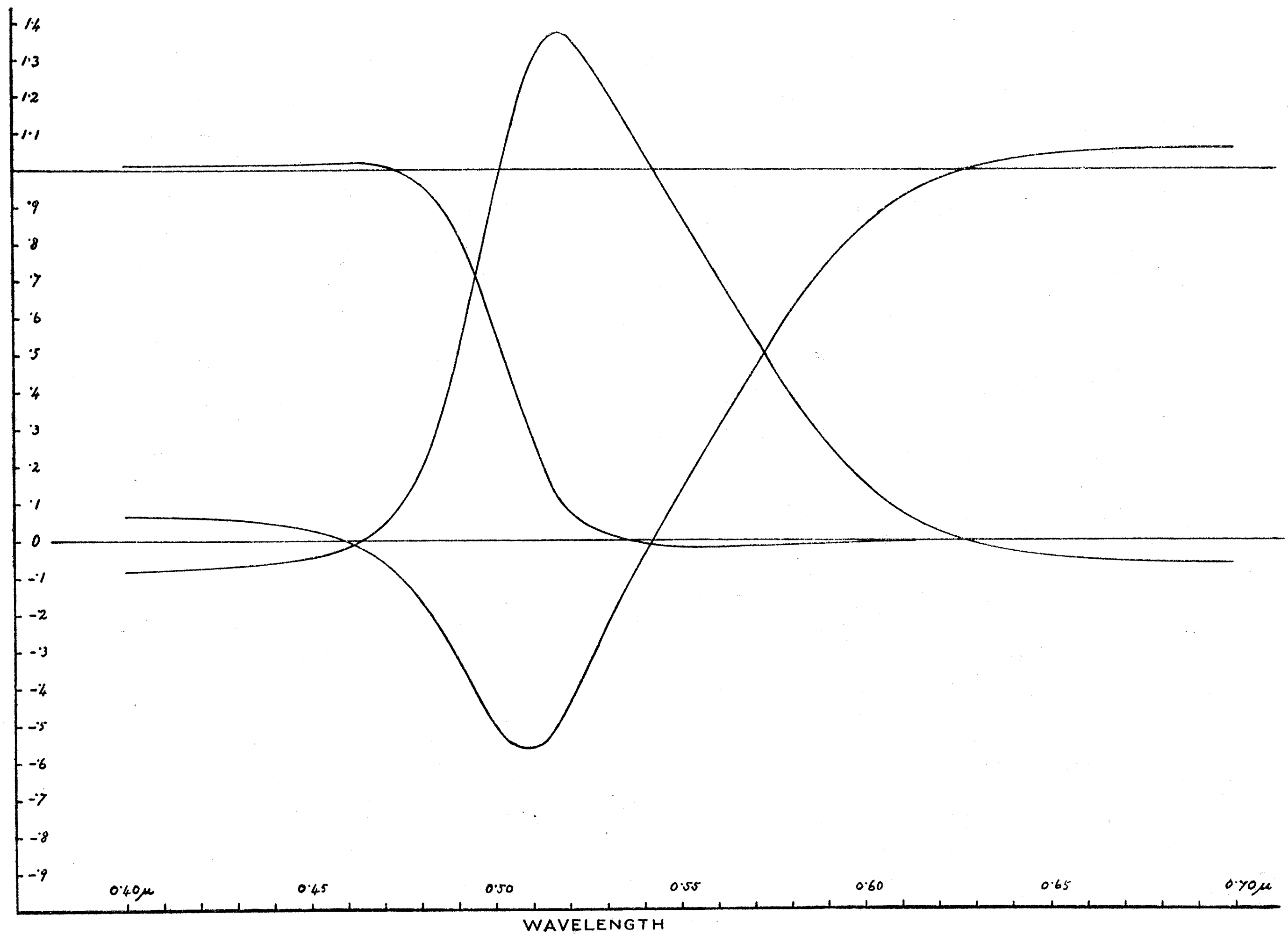

FIG. 4.

measured from suitable datum points. These were plotted in a series of graphs, and smooth lines put through them to remove obvious irregularities. From such curves corrected positions were found for the various interpolated points. The location of these on the triangle gave the trichromatic coefficients, and enabled the coefficient curves to be filled in with certainty in the regions between the observations.

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This method of interpolation is valuable for revealing discrepancies which cannot be detected from the coefficient curves themselves.

By the foregoing means I obtained for the working primaries a complete set of coefficients, at intervals of 0·001 μ, which accurately fitted the observations, and which varied in a proper manner in the intermediate regions. The values given in Table II (p. 180) are taken from this set.

Luminosity Factors.

The trichromatic unit, while it is the basis of the quantitative relations involved in colour-mixture calculations, is a purely arbitrary conception, and gives a measure of quantity in units peculiar to each stimulus, or rather to all stimuli which have the same colorimetric quality. In many practical problems the initial quantitative data are necessarily expressed in quantity units which are common to all stimuli; that is to say, in units of luminosity. To make it possible to change from luminosity units to trichromatic units, and vice versa, it is necessary to know the relative luminosities of trichromatic units of the primaries.

The luminosity factors were determined for the working primaries of the colorimeter by the following method. Three filters were obtained of the same colour as those constituting the primaries of the instrument and their transmission factors were determined, as presently to be described, for incident light from a gas-filled lamp at 2900° K.

Light from such a lamp was then thrown on the magnesium oxide screen, J, fig. 1. One of the filters, say the red, was placed in front of the colorimeter window, and the brightness of the transmitted light was matched by means of the red primary alone. The other filters were treated in the same way, the intensity of the illumination on the screen J being kept constant throughout. From the three scale readings thus obtained, and the known values of the relative luminosities of the light transmitted by the respective auxiliary filters, the relative luminosities *per scale division* of the three primary scales were obtained. As each measurement in this process consists of a brightness match between two identical stimuli the results are unaffected by the colour vision of the observer and give the true luminosity factors per scale division for any eye for which the assumed transmissions of the auxiliary filters are correct.* It is not, however,

* Accurate identity between the auxiliary filters and the primaries of the instrument is not essential. It is brightness only that is being matched, and a slight colour difference in the field of view is immaterial provided it is not enough to reduce the sensitivity of the settings. If the colour differences are sufficient to prevent satisfactory brightness matches with only one primary at a time, the colour can be equalised by introducing small amounts (positive or negative) of the other primaries, as required. The luminosity contributions of these are easily eliminated from the results. The auxiliary filters ought not, however, to differ greatly from the primaries, particularly in the case of the blue, otherwise the luminosity of the blue primary itself is swamped in that of the others and accurate results are impossible.

the luminosity factors per scale division of the instrument which we require for the present purpose, but the factors for trichromatic units of the primaries. This was obtained by dividing the luminosity factors per scale division by the mean white-light factors for the seven observers.

The method is extremely easy and accurate when once the transmission factors of the auxiliary filters have been determined. This, however, presented some difficulty. The straightforward procedure, if the visibility curve of the seven observers had been known, would have been to calculate the transmission factors from this visibility curve and the spectrophotometric curves of the filters. It was not feasible, however, to obtain the actual visibility curve of the observers* and it was intended to assume, as a first approximation, that their mean visibility curve was normal. Unfortunately the regions of the spectrum involved, in the case of the red and blue filters, are those for which the standard visibility data are of doubtful accuracy as representing the normal eye. A computation of the transmissions, using the standard data, gave the following values :—

	Actual. Per cent.	Relative.
Red filter	1·488	1·00
Green filter	5·233	3·51
Blue filter	0·179	0·120

These gave luminosity factors for the primaries in the ratio $L_R : L_G : L_B = 1 : 2{\cdot}60_6 : 0{\cdot}171$.

These factors, when employed in the normal use of the colorimeter, gave luminosity results which were inconsistent with other evidence, and the value for the transmission of the red filter was suspected of being in error. This was recalculated, using the visibility data for the red end of the spectrum given by HYDE, FORSYTHE and CADY.† This gave the transmissions of the filters in the ratio 1 : 3·93 : 0·13 and luminosity factors for the primaries of 1 : 2·92 : 0·18. The discrepancy of over 10 per cent. in the red-green ratio is serious, and a direct determination of the relative transmissions of the auxiliary filters, for the 2900° K source, was made by Dr. A. F. A. YOUNG and the writer, using a flicker photometer‡ under the standard conditions for flicker photometry. Although our visibility curves were not known, the mean of our ratios for the standard yellow and blue test solutions used to check observers for hetero-chromatic photometry was within ½ per cent. of the normal value. The relative trans-missions determined by us were 1 : 3·88 : 0·12, giving luminosity factors for the primaries in the ratios 1 : 2·88 : 0·17. This red-green ratio agrees to within 1½ per

* See Appendix I.

† 'Astrophys. J.,' vol. 48, p. 65 (1918).

‡ 'J. Sci. Instruments,' vol. 1, p. 182 (1924).

cent. with that based on HYDE, FORSYTHE and CADY'S visibility data, and bears out the conclusions of other workers that for the special purpose of computing the transmissions of deep red filters these data are more representative of the average eye than are the standard data.

The luminosity factors based on these measurements, when used in the ordinary routine of colorimetry, gave results which were consistent with all other available evidence. A more severe check on them was obtained in the process of calculating the trichromatic distribution curves of the spectrum, as described in a later section. To obtain numerical consistency in these curves a slight reduction in the green factor from 2·88 to 2·86 had to be made, and the values finally arrived at for the luminosity factors of the working primaries were, therefore

$$L_R : L_G : L_B = 1 : 2{\cdot}86 : 0{\cdot}170$$

It is usual to express these factors on such a numerical basis that their sum is unity. For their practical utilisation in colorimetric work we have found it to be more convenient to keep one of them, say the red, as unity. This reduces by one-third the numerical work involved in using the factors.

By multiplying the trichromatic coefficients of each unit equation throughout the spectrum by these factors, and adding, we obtain the luminosity factor, ${}_uL_\lambda$ for a trichromatic unit at each wave-length. It is useful to have these tabulated once and for all, as their use effects considerable simplification in many colour problems, such, for example (*loc. cit.* 1924–25), as the computation of the hue wave-length and colorimetric saturation corresponding to any stimulus.

The corresponding luminosity factor of the standard white for which the unit equation is, by choice of units, $(1/3)R + (1/3)G + (1/3)B$, is equal to a third of the sum of the luminosity factors of the primaries, that is, in the present case, $L_w = 1{\cdot}34$. By preparation of the data in this way much of the arithmetical work common to a large number of colorimetric computations is done once and for all.

Transformation to Standard Reference Primaries.

The results presented in the previous sections, being referable to the working primaries of the particular colorimeter employed in their determination, are not in a suitable form for general use. They must be transformed to a system of which the primaries are susceptible of simple and unambiguous definition without reference to any particular instrument. Much diversity of opinion exists as to the most suitable system of reference primaries. In order to keep this paper free from matter not relevant to the treatment of the results, I shall not discuss this problem here, but shall simply use, for the present purpose, the primaries adopted at the National Physical

Laboratory as standards for colorimetric specification.* These primaries are monochromatic radiations of the following wave-lengths :—

$$\begin{aligned} \text{Red} &= 0\cdot700\ \mu. \\ \text{Green} &= 0\cdot5461\ \mu. \\ \text{Blue} &= 0\cdot4358\ \mu. \end{aligned}$$

The unit equations of these primaries, expressed in terms of the working primaries, were found by interpolation of the values in their neighbourhood to be

$$\begin{aligned} R &= 1\cdot0604\ R_w - 0\cdot0604\ G_w + 0\ B_w. \\ G &= 0\cdot600\ R_w + 0\cdot9552\ G_w - 0\cdot0152\ B_w. \\ B &= 0\cdot0492_5\ R_w - 0\cdot0638_5\ G_w + 1\cdot0146\ B_w. \end{aligned}$$

where R, G and B now refer to the standard reference primaries, and R_w, G_w and B_w refer to the working primaries.

From these equations the transformation equations for converting data from the $R_w\ G_w\ B_w$ system to the R G B system were calculated by the method described in reference (*loc. cit.* 1924–25). These are :—

$$\begin{aligned} R_w &= 1\cdot1315\ R + 0\cdot0510\ G + 0\cdot0009\ B. \\ G_w &= -\ 0\cdot0720\ R + 0\cdot8951\ G + 0\cdot0156\ B \\ B_w &= -\ 0\cdot0595\ R + 0\cdot0539\ G + 0\cdot9835\ B. \end{aligned}$$

Applying these equations to the data of Table II, the values in Table III, columns 2, 3 and 4, were obtained in terms of the standard reference primaries.

In fig. 5 the results are shown in the colour triangle, using rectangular co-ordinates. The advantages of this form of colour chart, as compared with the traditional equilateral triangle employed by MAXWELL, have been discussed elsewhere. It is now almost universally used for the graphical representation of colorimetric results.

In a diagram of this kind we may choose any pair of the primaries as co-ordinates. G and B are chosen in fig. 4, as this gives a less elongated shape to the spectrum locus than either of the other arrangements.

* A particular trichromatic system has been adopted for the last few years at the National Physical Laboratory as a standard reference system for colorimetric specification.

The "fixed points" of this system are determined by the three monochromatic radiations of wave-lengths 0·700 μ, 0·5461 μ and 0·4358 μ, and the heterochromatic stimulus provided by the N.P.L. standard white light.

The considerations on which this system was adopted are discussed, and complete particulars of the N.P.L. standard white are given in another paper ("On the 'Fixed Points' of a Colorimetric System," in course of publication), and need not be repeated here. It is only necessary, for the purposes of the present paper, to give the properties of the white light, as listed in Table I. Column 3 gives the ordinates of the energy distribution curve, based on the best experimental evidence at present available, and column 4 the ordinates of the luminosity curve, assuming the international standard visibility data, which are given for reference in column 2.

It is also advisable to use a scale for the blue primary twice as open as that for the green. This enables some of the important features of the spectrum locus to be exhibited more advantageously without unduly increasing the size of the whole diagram.

By introducing the luminosity factors of the working primaries into the transformation relationships the corresponding factors for the standard primaries are found to be in the ratio

$$L_R : L_G : L_B = 1 : 4 \cdot 41 : 0 \cdot 052.$$

Applying these values to the transformed trichromatic coefficients, we obtain the values

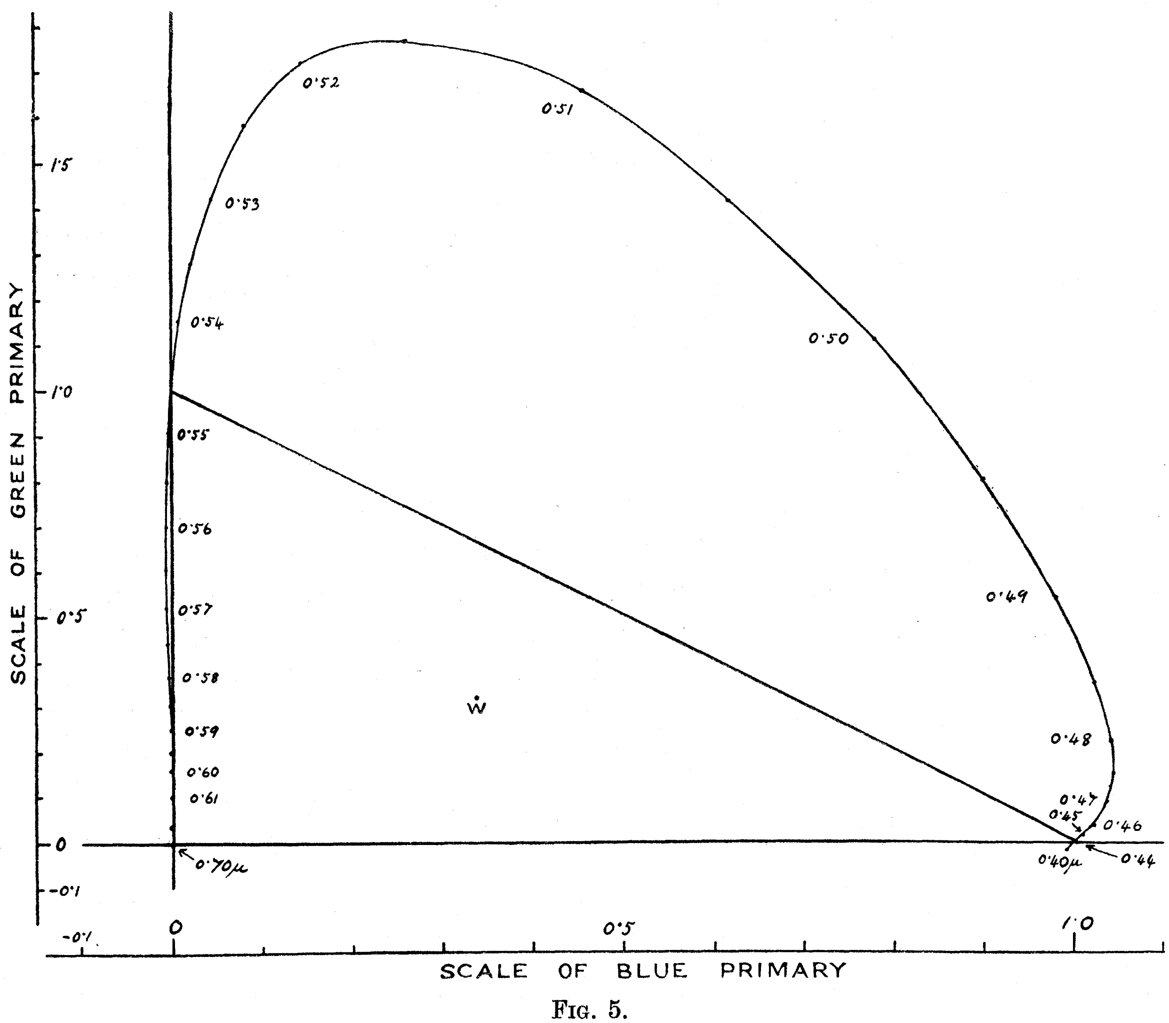

Fig. 5.

of ${}_{u}L_{\lambda}$ applicable to the new system of primaries. These are given in column 5, Table III (p. 182). The factor for white, in this system, becomes $L_w = 1 \cdot 82_1$.

The locus of the spectrum on the colour chart, together with the luminosity factors of the primaries, completely defines the eye with respect to those properties which determine the colour-match relations of all stimuli, whatever their spectral constitution. From the geometrical properties of the chart the effect of combining any number of constituent stimuli can be calculated. The problem of calculating the colorimetric quality of a heterochromatic stimulus from the distribution of energy in its spectrum is simply a special case of the general colour-mixture problem, since any such stimulus can be regarded as a synthesis of a number of sensibly monochromatic constituents. The relative intensities of these elementary constituents may be converted from an energy basis to a luminosity basis by means of the visibility curve, and then converted to the quantity basis appropriate to the colour chart, that is to say, to trichromatic units, by means of the luminosity factors. These processes are purely arithmetical, and, when the spectral distribution of the stimulus has been obtained in such units, the colorimetric quality may be obtained by the successive application of NEWTON's law to the mixture of the elementary constituents. The application of this procedure to a stimulus comprising a large number of monochromatic constituents is, however, extremely tedious. It is only mentioned to show that the computation of colorimetric quality from spectro-photometric data involves no special principle.

The work involved in such calculations can be enormously reduced by an initial combination of the colorimetric data with the visibility data to obtain the spectral distribution curves of the primaries.

The Spectral Distribution Curves of the Primaries.

In these curves the ordinates at any wave-length are in the proportions determined by the trichromatic coefficients for that wave-length, but their sum is proportional to the quantity, in trichromatic units, comprised within an infinitesimal wave-length band, centred at that wave-length, in the spectrum of a stimulus of known spectral energy distribution.

The curves of this type are usually termed the "mixture curves" of the spectrum for the primaries in question, but this term is unsuitable as it could equally well be applied to describe the trichromatic coefficient curves of fig. 4, and often is used in that sense. In a recent paper* Mr. DEANE B. JUDD has introduced the general term "distribution curves" to replace the various names hitherto used. This term seems much more suitable than any of the others in general use.

The spectral distribution curves based on the foregoing colorimetric measurements were worked out in the first instance from the data as given in Table II. They could have been worked out equally well directly from the transformed data of Table III, but it was considered preferable to complete the calculations in terms of the working primaries, as any discrepancies revealed in the process would be more easily related back to the original observations than if the transformed data had been used.

The procedure adopted was designed to obtain from the colorimetric observations a set of spectral distribution curves of the primaries such that their luminosity summation

* 'Bur. Stds. Research Paper,' No. 163 (1930).

would *accurately* fit the standard visibility data. Although, as pointed out in an earlier section, the standard data do not fit the present group of observers, and almost certainly do not represent any reasonably "normal" eye, for such purposes as computing the transmissions of red glasses in which the extremities of the spectrum are of relatively great importance, they are generally agreed to be a very close approximation for all other purposes, and for such purposes should fit closely any moderate-sized group of observers, chosen at random. This assumption might be unsafe for a group of only seven if there were no check on its validity. The various criteria which the spectral distribution curves should fulfil do, however, provide such a check.

The method of calculation was as follows. The ordinate of the luminosity curve of the white light (see Table I) at any wave-length λ, was divided by ${}_uL_\lambda$, the luminosity factor of a trichromatic unit at that wave-length. The quotient, *n*, is a measure of the relative intensity of the spectrum at this wave-length, expressed in trichromatic units. Then if the coefficients of the unit equation are *a*, *b* and *c*, the ordinates of the distribution curves are *na*, *nb* and *nc*.

This process is essentially identical with that of splitting up the luminosity ordinates among the three primaries in proportion to their luminosity values, but in practice effects some saving in arithmetical work as compared with the direct application of that process.

The areas under the three distribution curves represent the relative contributions of the primaries in matching the standard white and should be equal. If any inconsistency exists in the initial data the areas will not be equal, and the error must be sought for in one or other of the following causes :—

(*a*) Inconsistency between the assumed visibility curve and that which actually fits the observers.
(*b*) Errors in the trichromatic coefficients.
(*c*) Errors in the determination of the luminosity factors of the instrument primaries.

When the curves were calculated with the experimentally determined luminosity factors of $L_R : L_G : L_B = 1 : 2 \cdot 88 : 0 \cdot 17$, the areas were as nearly equal as mattered in practice, but in order to get a higher order of arithmetical consistency, they were recalculated with the slightly reduced values of $2 \cdot 858$ for L_G, and $0 \cdot 169$ for L_B, which gave areas as nearly equal as could be determined by careful planimetry.

The curves were then transformed to the reference primaries and are given in Table III for the N.P.L. standard white light, and also for the equal-energy spectrum, in which form they are best adapted for general work. These equal-energy curves give a luminosity summation identical with the standard visibility curve.

The conclusion to be drawn from the fact that the curves fulfil the three criteria of fitting the experimentally determined trichromatic coefficients; fitting the normal visibility curve; and having luminosity factors for the primaries which agree with the independently determined values for these factors as closely as such determinations can at present be made, is that the mean visibility curve of the seven observers is sufficiently close to the standard curve that their colorimetric results do not differ to a material extent from those of an eye accurately represented, as regards its visibility function, by that curve.

It does not prove complete accuracy either of the colorimetric data or of the assumed visibility curve. An infinite number of independent criteria would be required to establish this, since the experimental data may vary independently at every point.

But if the three specified criteria can be fulfilled, the data must be *very nearly* consistent in every respect, unless there are a number of quite fortuitous compensations in the errors. The latter state of affairs is sufficiently improbable to justify the presumption that the distribution curves obtained are a close approximation to the true curves pertaining to the same normal eye as is defined by the standard visibility data. It is important to point out, however, that in the absence of the check afforded by the independent measurements of the luminosity factors of the primaries, no such conclusion could have been drawn. It is possible to fit any set of trichromatic coefficients, within reason, to any assumed visibility curve by choosing suitable values for the luminosity factors of the primaries. This is easily done in a few trials by successive approximation, and values for $L_R : L_G : L_B$ are obtained in the process which give an exact luminosity summation to the assumed visibility curve. But these luminosity factors may be quite fictitious, and may not represent the proportions by luminosity, in which the three primaries have to be combined to match white. This is especially the case with the blue primary. Owing to the relatively low luminosity of this primary as compared with the red and green, and the manner in which the positive and negative values of these two primaries appear in the blue and violet region of the spectrum, a small error, either in the trichromatic coefficients in this neighbourhood, or in the assumed visibility curve in the neighbourhood, may make a large difference in the value of L_B which is required to give the desired luminosity summation. In fact, errors in the trichromatic coefficients in the blue and violet region, which would be sub-limenal as regards colour discrimination, may produce large errors in the luminosity factor to be attributed to the blue primary, and would give distribution curves which were quite unreliable for computations involving stimuli which have localised concentrations of energy in different parts of the spectrum. Thus it is no proof of the accuracy of a set of spectral distribution curves that luminosity factors can be found which give a luminosity summation approximately or even identically fitting the normal visibility curve. This only means that they will give correct results for the white light on which they are based by definition of units, or for lights not differing too much from this in spectral energy distribution. Additional criteria, based on the quantitative relations of stimuli which are either monochromatic or at least of high colorimetric saturation, must be fulfilled. The comparison of the values of $L_R : L_G : L_B$ which are required in order to fit the colorimetric data to the visibility data with their true values, as obtained independently, is one such check, and is probably the easiest to apply.

Comparison with other data.

Reference was made in the introduction to the results recently published by W. D. WRIGHT (*loc. cit.*, 1928–30). WRIGHT's results refer to ten observers, and his experimental conditions, as regards field size and the white light used to evaluate his units, were the same as those employed at the National Physical Laboratory. The uniformity in these two essential conditions renders WRIGHT's results strictly comparable with those obtained in the present investigation. In all other respects the experimental conditions differed in the two investigations. WRIGHT employed a colorimeter (*loc. cit.*, 1927–28) in which the primaries were obtained spectroscopically and which differed completely in all points of manipulation and calibration from the writer's colorimeter and the auxiliary apparatus of fig. 1, and a comparison of the results should reveal any serious errors due to systematic causes which affected either determination.

In making a comparison it is preferable to consider the unit equations and their loci on the colour chart rather than the spectral distribution curves of the primaries, as it is in the unit equations that the colorimetric work lies. WRIGHT's results were first published in terms of his own working primaries, but in the discussion of his second paper (*loc. cit.*, 1929–30) he tabulates them in terms of the N.P.L. reference primaries. It is also of interest to compare the results worked out by WEAVER from the data of ABNEY and KÖNIG, which were recommended for general use by the Colorimetry Committee of the Optical Society of America (*loc. cit.*, 1922).

To render these comparable with the others it is necessary to transform the data from WEAVER's primaries to the N.P.L. reference primaries, and also to change the units so that they refer to the N.P.L. white instead of "Abbot noon sunlight." The process employed was as follows. From a large scale graph of WEAVER's coefficients the unit equations of the N.P.L. primaries were found to be :—

$$\begin{aligned} R &= 1\cdot000\,R' + 0\,G' + 0\,B' \\ G &= 0\cdot391\,R' + 0\cdot588\,G' + 0\cdot021\,B' \\ B &= 0\,R' + 0\cdot004\,G' + 0\cdot996\,B' \end{aligned}$$

where R′, G′ and B′ denote WEAVER's primaries. From these the transformation equations for transforming from the R′, G′, B′ system to the R, G, B system were calculated to be

$$\begin{aligned} R' &= +\,2\cdot962\,R + 0\,G + 0\,B \\ G' &= -\,1\cdot970\,R + 1\cdot0049\,G - 0\cdot037\,B \\ B' &= +\,0\cdot008\,R - 0\cdot0049\,G + 1\cdot037\,B \end{aligned}$$

These equations were used to transform both the unit equations and the ordinates of the distribution curves for the equal energy spectrum. The latter curves were required in order to effect the change of units corresponding to the different white lights. The data, as first transformed, refer to the N.P.L. primaries, but the units are such that the Abbot sunlight occupies the W point on the colour chart. To relate the values to the

N.P.L. white, the distribution curves for the equal energy spectrum were multiplied out by the energy ordinates of the N.P.L. white and replotted. The areas under the curves were in the ratio

$$R : G : B = 0 \cdot 366 : 0 \cdot 322 : 0 \cdot 312$$

These figures are therefore the coefficients in the unit equation of the N.P.L. white in the system defined by WEAVER's distribution curves, the N.P.L. standard reference primaries, and the Abbot sunlight.

The change of units required to put the N.P.L. white at the W point of the chart is effected by multiplying the coefficients of R, G and B in the transformed unit equations by the respective factors

$$0 \cdot \dot{3}/0 \cdot 366 \; ; \; 0 \cdot \dot{3}/0 \cdot 322 \; ; \; 0 \cdot \dot{3}/0 \cdot 312$$

i.e.,

$$0 \cdot 911 \quad ; \quad 1 \cdot 035 \quad ; \quad 1 \cdot 068$$

The new coefficients thus obtained are in the correct proportions, but will not, in general, add up to unity. By dividing throughout by their sum the new unit equation is obtained.

By these means we obtain WEAVER's data on the system defined by the N.P.L. reference primaries and the N.P.L. white, on which basis they may be compared with the determinations of WRIGHT and the author.

The three sets of data are plotted together on the colour chart in fig. 6. The dots refer to the N.P.L. values, and the crosses and circles to those of WRIGHT and WEAVER. It will be observed at a glance that there is very close agreement between the N.P.L. values and WRIGHT's. It is impossible, on the scale of fig. 6, to draw separate loci for them from the red end of the spectrum to $0 \cdot 535\,\mu$. The spacing of the points along the locus is also in quite close agreement. The wave-length displacement along the common part of the locus nowhere exceeds the hue-discrimination limen.*

From $0 \cdot 53\,\mu$ the loci definitely diverge until they converge again at the blue primary. The apparent divergence is, however, much exaggerated in this part of the diagram. We have to interpret it in terms of colour difference. The displacements along the curves nowhere exceed about $0 \cdot 001\,\mu$, which again is less than the hue-discrimination limen. As regards the separation of the loci, this is greatest at about $0 \cdot 50\,\mu$. If we draw a line from W through the point $0 \cdot 50\,\mu$ on the N.P.L. locus, it meets WRIGHT's locus at about $0 \cdot 499\,\mu$. The displacement along the radial line corresponds to a difference in colorimetric saturation of $2\frac{1}{2}$ per cent. There is no very precise published data on colour discrimination in directions normal to the locus. From the observations of JONES and LOWRY†, the saturation discrimination limen in the neighbourhood of the locus is between 2 and 3 per cent. Less than this can be detected in the mean of a

* GUILD, 'Proc. Opt. Convention,' part 1, p. 61, fig. 8 and Table 5 (1926).

† 'J. Amer. Opt. Soc.,' vol. 13, p. 25 (1926).

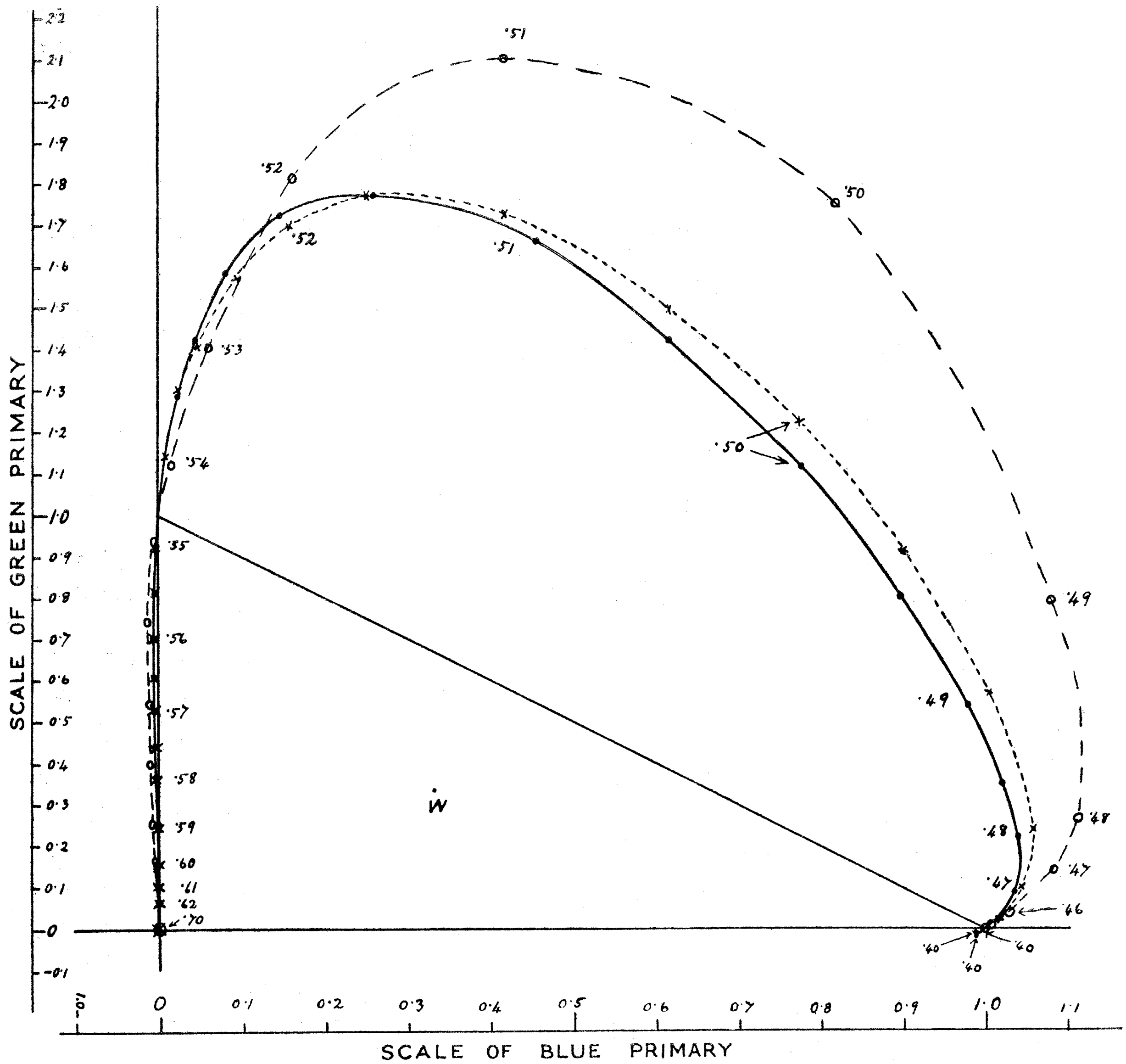

FIG. 6.

series of colour measurements, but $2\frac{1}{2}$ per cent., if not actually sub-limenal, is certainly not much beyond it. Thus the greatest differences exhibited by the two sets of data from the red extremity of the spectrum to the neighbourhood of the blue reference primary, when interpreted in terms of colour discrimination, are either definitely negligible or near the border line. This does not mean that the differences have no significance. From the means of a large number of observations colorimetric values can be determined within very much narrower limits than are indicated by the discrimination limen in their neighbourhood, and the difference between the two curves either represents a real, though small, difference in the colour-vision of the two groups of observers, or a systematic difference introduced by a slight difference in the standard white light as set up in the two laboratories, or by some other instrumental discrepancy. But the differences, from whatever cause arising, are negligible in the sense that a colour match made by an observer accurately represented by one set of data would not be considered wrong when looked at by an observer represented by the other set.

The details at the blue end of the spectrum are too congested in fig. 6 for proper examination. In fig. 7 this region is shown, for the two modern determinations, on a much larger scale. Several interesting features may be noted. Both determinations show that a quite definite and progressive change in colour takes place right to the

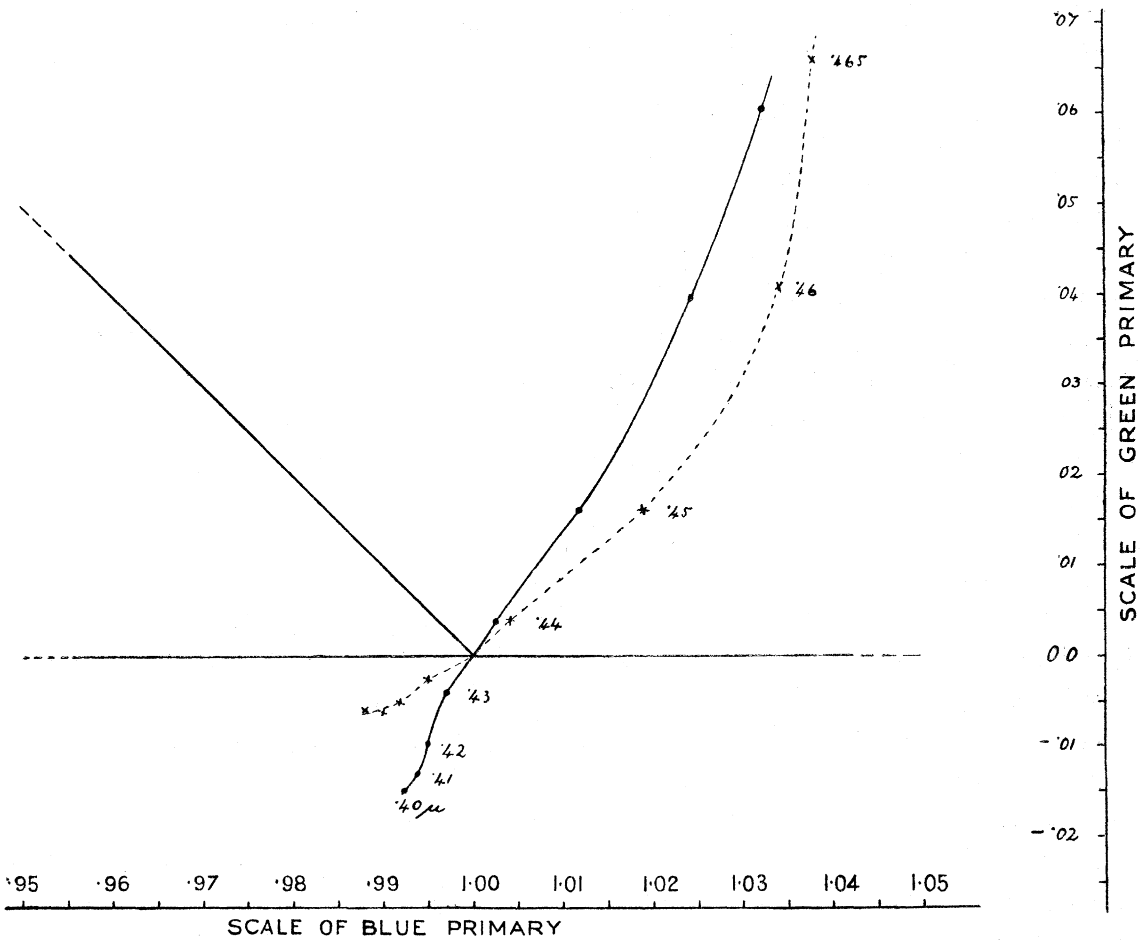

Fig. 7.

violet extremity of the visible spectrum. The distribution of the points from the blue primary to 0·40 μ is very similar in both loci.

This feature is one on which some uncertainty has existed, some writers holding the view that there is a constant colour range in the violet as there is at the red end of the spectrum. The progressive change shown by the König-Ives data has been attributed to errors arising from stray light. Such an assumption is untenable as an explanation of the form of the loci in fig. 7. Apart from the precautions taken in both investigations to exclude stray light, neither curve exhibits the sharp curl inwards towards white which would be caused by appreciable impurity in this part of the spectrum.

A second feature of both curves is the slight inward bend at $0 \cdot 43\ \mu$. When this feature was observed in the N.P.L. curve, I attributed it to experimental error, as the displacement from a curve without this bend is completely sub-limenal. The fact, however, that the same phenomenon, despite its slightness, emerges from the results of the two separate investigations, suggests that it may be real, for I am unable to account for it by any peculiarity in experimental conditions which could be common to both investigations. The wave-length at which it occurs was, of course, in a different relationship to the working primaries in the two investigations, and the actual colour matches involved in determining this point on the locus were quite dissimilar in the two cases.

As regards the divergence of the two loci, this would be sub-limenal in a single colour match except, perhaps, towards the extreme end of the spectrum.

The general conclusion to be drawn from this comparison is that WRIGHT'S results and the author's are sensibly identical, in the sense previously indicated that if we regard each set of data as accurately defining the properties of two hypothetical observers, neither observer would be able to disagree definitely with a colour match adjusted by the other. At most parts of the spectrum they would be absolutely satisfied by the same match, and in regions where disagreement is greatest they might, at the worst, be doubtful. This applies to the severe test of colour matches in which the test colour is monochromatic. In all cases involving less saturated colours, the discrepancies become of correspondingly less importance.

WEAVER'S curve, on the other hand, differs materially from the two others. The closest agreement is in the red to green region. Here the discrepancies, though considerably greater than those between WRIGHT'S data and the author's, are not excessive. In the green to blue region the disagreement is very pronounced. Here, again, it has to be pointed out that the apparent difference is over-emphasised by the diagram, and that a true comparison can only be made on the basis of the colour difference corresponding to the displacements. It is clear, however, that throughout this region the average departure of the curve from the other two is about five or six times as much as their own separation, the ratio being in some parts much greater than this.

This agrees with our experience at the Laboratory when using the KÖNIG-IVES data for colorimetric computations. These data were found to give values for saturated colours from red to green which were in quite good agreement with experimental colour matches of various kinds; but for colours of other hues, unless these were of comparatively low saturation, serious discordance was found. WEAVER'S values are, of course, only partly based on KÖNIG'S data, and partly on ABNEY'S, and it is claimed for them by PRIEST, GIBSON and MUNSELL* that they give a closer approximation to

* 'J. Amer. Opt. Soc.,' vol. 8, p. 28 (1928), summary.

experimental measurements by the monochromatic-plus-white method than the König-Ives set. Judd (*loc. cit.*, 1930) has computed, by the method of least squares, the best value of the luminosity factors of the primaries, and shows that the distribution curves, when evaluated by these factors, summate to a visibility curve which differs from the standard curve by amounts which " are negligible in comparison to differences between individual visibilities."

The improvement, as compared with the König-Ives data, is not very pronounced. Of the eight comparisons published in reference 20, Weaver's data gives a better value of the hue wave-length in five cases, and König-Ives in three. As regards colorimetric saturation (" Purity " in the American literature), the König-Ives values are nearer the experimental result in five cases, Weaver's values in two, and in one case they gave identical results.

None of the eight comparisons relate to stimuli of hue wave-length between $0 \cdot 49\ \mu$ and $0 \cdot 53\ \mu$, which is the region of the chart where the discrepancies are greatest. In any case, however, comparisons of experimental results and those computed from the energy distribution of the stimuli are not an adequate substitute for a direct check on the spectral relationships themselves, because the inaccuracies of the spectral data become of decreasing importance as the extent of the spectrum comprised within the stimulus increases. The best check on the spectral data consists of an experimental comparison of the colour match relations of spectral colours with those predicted by the application of Newton's law to the locus on the colour chart. Experiments of this kind readily demonstrate that the bulge in the blue-green region of either König's locus or Weaver's is excessive for any ordinary observer.

As regards the spectral distribution curves, I have already pointed out that a fit with the visibility curve, whether partial or complete, is no proof of consistency, unless the luminosity factors attributed to the primaries are correct, in accordance with some independent check. In the original publication of Weaver's data (*loc. cit.*, 1922) values are suggested for these factors which, when transformed to the N.P.L. reference primaries, would make the luminosity factors for these primaries $1 : 6 \cdot 8 : 0 \cdot 098$.

The values computed by Judd, when similarly adapted to the N.P.L. primaries, are $1 : 4 \cdot 84 : 0 \cdot 066$. The latter are much nearer the values obtained from the N.P.L. determination, namely, $1 : 4 \cdot 41 : 0 \cdot 052$, but differ from them by amounts which are large compared with any possible uncertainty in the experimental work on which they are based.

Thus Weaver's data do not fulfil the condition of quantitative self-consistency even reasonably well.

A Proposed " Normal " Eye for Colorimetry.

Determinations of the type herein described fulfil two functions. They are, in the first place, additions to our quantitative knowledge of the behaviour of the human eye. From this point of view there can be no question of standardising data; no question of *defining* any set of experimental results as "true," or as representing the performance of the average eye. Every new group of observed facts must be put in its proper relationship to those already known in order to build a structure of knowledge concerning the phenomena of vision which becomes more comprehensive in scope and more secure in its foundations with every accession of properly accredited facts. The experimental results given in the present paper are presented, and their relations to others discussed, simply as a contribution to the growing structure, to be verified, modified or superseded as and when other information becomes available.

There is, however, a utilitarian aspect of such determinations which makes it desirable to standardise certain data for use in technical and industrial work. The international photometric scale which governs the output of large industries concerned with the production of illuminants, necessitates the elevation of some particular set of visibility data to the dignity of a standard, to be used universally in all computations carried out for technical purposes. Such standards do not necessarily stand for all time, but remain fixed until such time as they may be altered by the decision of properly representative bodies in the light of such new information as may have accrued since the adoption of the standard. Hopeless confusion would arise if every lamp manufacturer, or every photometric standardising laboratory, employed units based on individual judgment as to the most accurate visibility data available at any particular time.

In the same way a standardised set of trichromatic data, to be accepted as representing the "normal" eye in technical colorimetric work, has long been overdue. No such standard has hitherto been adopted because of the extreme paucity of the data on which to base one. Such standardisation cannot usefully be effected until data are available which can be regarded as sufficiently near the truth that they are not likely to be *seriously* modified by future determinations. The new results of the present paper, taken in conjunction with those of WRIGHT, indicate that this stage has been reached, and that a "normal" eye for technical colorimetric work can now be adopted as a standard, with the same status, and limitations as to permanence, as the International visibility curve.

The number of observers covered by these two investigations is not large, but the fact that they were tested in two groups, in different institutions, and by different experimental methods, and that the results of the two groups agree so remarkably well, gives an authority to the two investigations which neither could have claimed separately, even had the same total number of observers, or more, been involved; and shows that when proper care is taken to obtain suitable experimental conditions, and to employ properly standardised auxiliary equipment, a close approximation to the properties of the average eye may be obtained from a comparatively small group of observers. The two groups, as has been shown, are in agreement for all practical purposes, and I suggest that the mean of their results are suitable for standardising as the "normal" colorimetric data for technical purposes to correspond with the standard "normal" visibility data. I have given serious consideration as to whether any use can profitably be made of the KÖNIG-ABNEY figures, or any part of them, in giving greater weight to a proposed "normal" set, but have concluded that these can only be regarded as of historical interest, and of no value *for this purpose.* This conveys no reflection on the work of these great pioneers in the metrology of vision. It has to be borne in mind that neither KÖNIG nor ABNEY carried out their investigations with the object of providing precise numerical data for technical needs, which were, indeed, non-existent when their work was carried out. In both cases the object of their researches was concerned with the theory of the visual process; and their methods of experiment, and methods of reducing their figures from the combined results of normal and abnormal subjects, were determined by that object, and involved approximations both of an experimental and theoretical kind which are not admissible in work directly aiming at accurate numerical results. Not only so, but the experimental equipment of those days, and the precision with which such auxiliary constants as, for example, the reference white, could be specified, were naturally inadequate when judged by present-day requirements. These reasons are, I think, sufficient justification for disregarding the old work, in so far as the numerical results are concerned, and basing a provisional standard on the two investigations which have been directly concerned with obtaining precise data in accordance with modern experimental requirements.

In Table IV, columns 2, 3 and 4, are given the trichromatic coefficients which are the mean of the two sets. On the basis of the relative size of the groups, WRIGHT'S values should have been weighted in the ratio of 10 to 7. On the other hand, any part of the difference which may be due to incomplete identity in the auxiliary standards employed is more likely to be associated with WRIGHT'S values, owing to the greater facilities for the accurate maintenance and checking of such standards at the National Physical Laboratory. Putting these two opposing considerations against each other an unweighted mean of the two sets of data seems most reasonable. The mean trichromatic coefficients are shown plotted in the colour chart in fig. 8.

In obtaining the spectral distribution curves, columns 6 to 11, Table IV (p. 185), I have not taken the average of WRIGHT'S distribution curves and my own. Owing to an oversight on my part, WRIGHT was not supplied with a statement of the actual energy distribution curve of the N.P.L. standard white light, and deduced his mixture curves from an assumed black-body distribution for a temperature of 4800° K. While the colour of this light is fairly close to that of the N.P.L. white, it is not necessarily identical with it. WRIGHT'S curves and the author's are, consequently, on a slightly different basis.

The distribution curves corresponding to the mean colorimetric data have therefore been worked out *ab initio*, by fitting the latter to the standard visibility data, by the same method as was described in connection with the author's own curves.

The luminosity factors of the reference primaries which were necessary to obtain com-

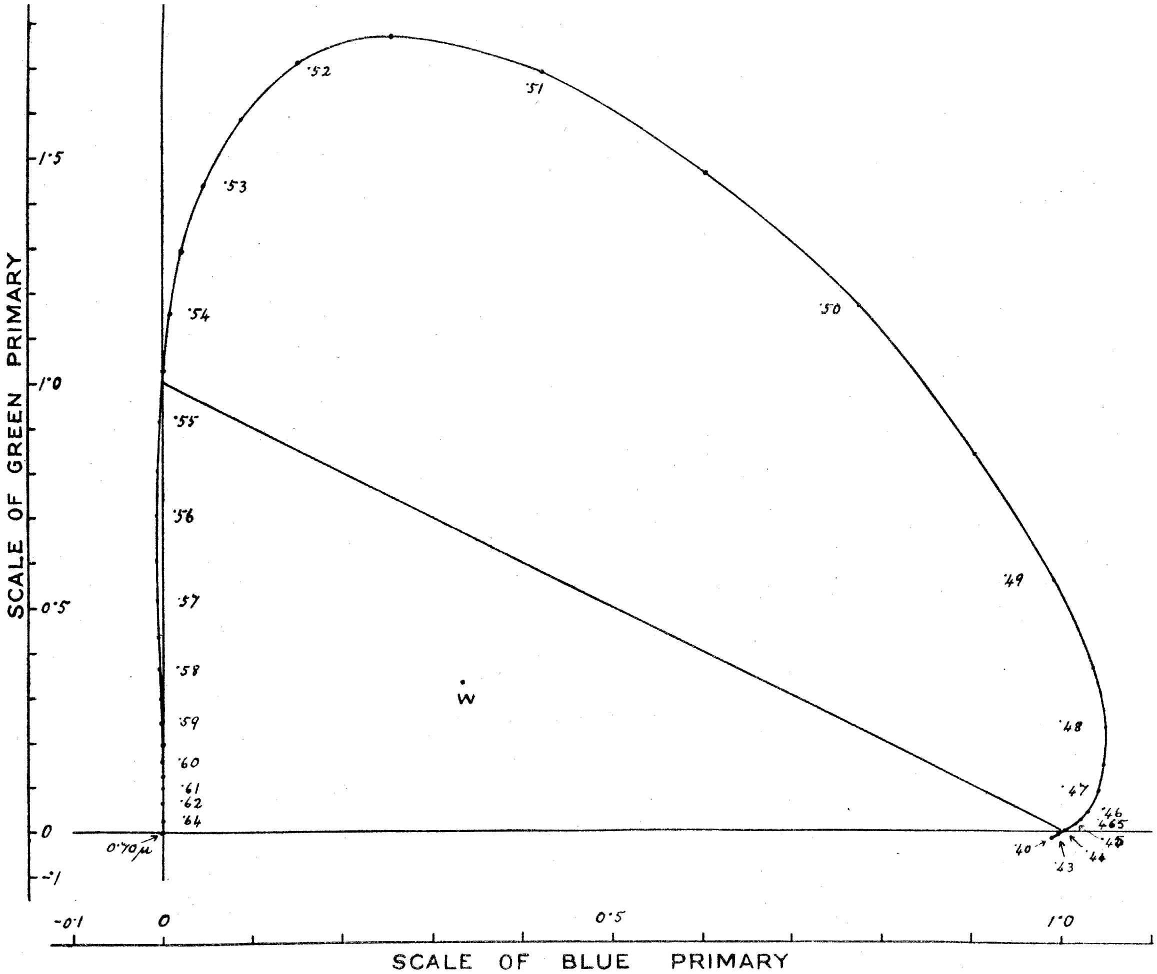

FIG. 8.

plete consistency in the mean data, were $L_R : L_G : L_B = 1 : 4{\cdot}39 : 0{\cdot}048$. These differ slightly from the values which fit the author's own data, and which were consistent with the independent determination of the factors of the working primaries of the Laboratory colorimeter.* Values more closely agreeing with these could have been obtained by neglecting WRIGHT's values between 0·44 μ and the end of the spectrum, but as the independent check of the luminosity factors was only made in one of the investigations, and as the discrepancy is small enough to be within the range of overall uncertainty in the values, it appeared preferable to retain the mean colorimetric data throughout, and to accept the new values for the luminosity factors.

The data of Table IV therefore fulfil the following conditions :—

1. The unit equations are a straight mean of those obtained by WRIGHT and the author.

* WRIGHT was unable to determine the luminosity factors of his own working primaries to his satisfaction, and so there is no check on the luminosity relationships obtainable directly from his work.

2. The spectral distribution curves, when weighted by the luminosity factors of the primaries, summate exactly to the standard "normal" visibility curve.
3. The luminosity factors are in reasonably close agreement with those determined by independent methods in the N.P.L. investigation.

Those conditions are not, of course, affected by expressing the data in terms of any other set of primaries, real or otherwise, or by altering the units so as to base their relative magnitudes on a standard white differing in colour from the N.P.L. white. The procedure involved in a change of reference white is simple. The ordinates of the distribution curves of the equal-energy spectrum are multiplied by the ordinates of the energy distribution curve of the new white and re-plotted. The relative areas under the R, G and B curves give the coefficients in the unit equation of the new white expressed on the old system. If these are α', α'' and α''', say, the factors by which the coefficients on the old system must be multiplied, in order to place the new white at the W point of the colour chart, are respectively $0 \cdot \dot{3}/\alpha'$, $0 \cdot \dot{3}/\alpha''$ and $0 \cdot \dot{3}/\alpha'''$. The figures first obtained in multiplying the coefficients of a unit equation by these factors will not in general add up to unity, but by dividing through by their sum the new unit equations are obtained. If the old luminosity factors of the primaries are $1 : L_G : L_B$ the new factors are $1 : \frac{\alpha'}{\alpha''} L_G : \frac{\alpha'}{\alpha'''} L_B$.

To change the spectral distribution curves to the new system of units we simply have to multiply the ordinates of each curve by the appropriate factor. These new curves will be of the correct shape and, when weighted by the new luminosity factors, will give a visibility curve of the correct shape. This is all that is required, the actual scale of the ordinates being immaterial for any practical purpose. The scale may, however, be adjusted to make the luminosity summation of the curves numerically identical with the visibility curve by multiplying all ordinates by a suitable factor.

The proposal to standardise the data of Table IV does not, therefore, necessarily carry with it the proposal to adopt either the N.P.L. system of reference primaries, or the N.P.L. standard white. The question whether the visual relations embodied in the data are suitable to represent a standard normal eye for technical colorimetry, may be, and ought to be, considered separately from those questions which are relevant to the choice of suitable reference standards.

In conclusion I have to acknowledge my indebtedness to Dr. A. F. A. Young, who, in addition to the assistance with the practical work acknowledged earlier, carried out the calculations of the spectral distribution curves for the working primaries of the colorimeter; and to Mr. H. G. W. Harding, junior observer in the Optics Division of the National Physical Laboratory, who performed all the numerical work of transforming the results to the reference primaries, of reducing the Weaver data to the same basis, and of computing the mean data of Table IV.

Appendix I.

The Trichromatic Determination of both Photometric and Chromatic Functions.

The method adopted in the present paper for obtaining the spectral distribution curves for a group of subjects, assumed to be at least *nearly* normal in the mean of their properties, by combining their experimentally determined trichromatic coefficients for the spectrum with an assumed normal visibility curve, is a simplification of a general method which may be employed for obtaining both the trichromatic coefficients and the visibility curve for any subject from one set of measurements carried out entirely on a trichromatic colorimeter.

The observations made by the subject are the same as those described in the present paper, being merely those involved in obtaining the trichromatic coefficients of the spectrum in terms of the working primaries of the colorimeter. The additional information required consists of a radiometric calibration of the apparatus providing the monochromatic illumination, in order that the relative energy value of the monochromatic radiation in the test field of the colorimeter may be known for each " set-up " employed in the series of measurements.

With this additional information it is possible, by starting with assumed values of the luminosity factors of the instrument primaries, to obtain, by successive approximation, the true values of these factors for the subject, and his own visibility curve which, when combined with his colorimetric values for the spectrum, will give a self-consistent set of primary distribution curves in accordance with the criteria discussed in the body of the paper.

The complete method was not employed for the present work because there was not available, at the time when the work was done, sufficiently sensitive radiometric equipment for evaluating the energy intensities.

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TABLE I.—N.P.L. Standard White Light.

λ = wave-length (in μ).

V_λ = ordinate of International Standard Visibility Curve; (I.C.I., 1924).

E_λ = ordinate of Energy Distribution Curve; values reduced so that E = 100 at λ = 0·56 μ

$V_\lambda E_\lambda$ = ordinate of Luminosity Curve. Values at every 0·005 μ interpolated.

λ.	V_λ.	E_λ.	$V_\lambda E_\lambda$.	λ.	V_λ.	E_λ.	$V_\lambda E_\lambda$.
0·38	—	28·0	—	0·60	0·631	$92 \cdot 1_5$	58·15
0·385	—	32·0	—	0·605	—	90·6	51·26
0·390	—	35·5	—	0·61	0·503	$89 \cdot 4_5$	45·00
0·395	—	39·0	—	0·615	—	88·7	39·28
				0·62	0·381	$88 \cdot 0_5$	33·57
0·40	0·0004	43·2	0·017	0·625	—	87·6	28·09
0·405	—	47·4	0·036	0·63	0·265	87·4	23·16
0·41	0·0012	$51 \cdot 5_5$	0·062	0·635	—	87·4	18·95
0·415	—	55·7	0·120	0·64	0·175	$87 \cdot 6_5$	15·34
0·42	0·0040	$60 \cdot 1_5$	0·241	0·645	—	$87 \cdot 8_5$	12·20
0·425	—	64·0	0·473				
0·43	0·0116	67·3	0·781	0·65	0·107	$87 \cdot 9_5$	9·41
0·435	—	70·1	1·161	0·655	—	$87 \cdot 9_5$	7·24
0·44	0·023	72·6	1·67	0·66	0·061	87·9	5·36
0·445	—	74·4	$2 \cdot 27_5$	0·665	—	87·7	3·85
				0·67	0·032	87·5	2·80
0·45	0·038	75·9	$2 \cdot 88_5$	0·675	—	$87 \cdot 2_5$	2·03
0·455	—	77·1	$3 \cdot 72_5$	0·68	0·017	$86 \cdot 9_5$	1·48
0·46	0·060	78·2	4·69	0·685	—	$86 \cdot 6_5$	1·04
0·465	—	79·2	5·87	0·69	0·0082	86·3	0·71
0·47	0·091	80·1	7·29	0·695	—	86·0	0·503
0·475	—	80·8	9·13				
0·48	0·139	81·2	11·29	0·70	0·0041	$85 \cdot 6_5$	0·351
0·485	—	81·5	13·91	0·705	—	85·2	0·251
0·49	0·208	$81 \cdot 6_5$	16·98	0·71	0·0021	84·6	0·178
0·495	—	$81 \cdot 5_5$	20·96	0·715	—	$83 \cdot 9_5$	0·121
				0·72	0·00105	83·2	0·087
0·50	0·323	80·9	26·12	0·725	—	$82 \cdot 6_5$	$0 \cdot 062_5$
0·505	—	$79 \cdot 7_5$	32·40	0·73	0·00052	81·6	$0 \cdot 042_5$
0·51	0·503	79·1	39·77	0·735	—	80·7	0·029
0·515	—	79·4	48·15	0·74	0·00025	79·7	0·020
0·52	0·710	81·1	57·57	0·745	—	78·9	0·013
0·525	—	83·9	66·47				
0·53	0·862	$86 \cdot 7_5$	74·77	0·75	0·00012	77·7	0·009
0·535	—	89·6	81·74	0·755	—	76·4	$0 \cdot 006_5$
0·54	0·954	$92 \cdot 3_5$	88·09	0·76	0·00006	75·1	$0 \cdot 004_5$
0·545	—	94·8	93·15	0·765	—	73·9	—
				0·77	—	72·7	—
0·55	0·995	97·1	96·62	0·775	—	71·3	—
0·555	—	98·9	98·81	0·78	—	69·8	—
0·56	0·995	100·0	99·50				
0·565	—	100·3	98·39				
0·57	0·952	100·1	95·30				
0·575	—	99·3	90·31				
0·58	0·870	97·9	85·15				
0·585	—	$96 \cdot 1_5$	79·29				
0·59	0·757	94·7	71·66				
0·595	—	93·6	64·80				

TABLE II.—Mean Trichromatic Coefficients of Seven Subjects tested at the National Physical Laboratory.

λ = wave-length (in μ).

${}_u a_\lambda$, etc. = coefficients in the expression ${}_u a_\lambda \mathrm{R} + {}_u b_\lambda \mathrm{G} + {}_u c_\lambda \mathrm{B}$, in which ${}_u a_\lambda + {}_u b_\lambda + {}_u c_\lambda = 1$. This represents the colorimetric quality of a monochromatic stimulus of wave-length λ in terms of R, G and B, which denote the working primaries of the instrument on which the measurements were made.

${}_u \mathrm{L}_\lambda$ = luminosity factor of one "trichromatic unit" of monochromatic stimulus of wave-length λ, $= {}_u a_\lambda \mathrm{L_R} + {}_u b_\lambda \mathrm{L_G} + {}_u c_\lambda \mathrm{L_B}$, where $\mathrm{L_R}$, $\mathrm{L_G}$ and $\mathrm{L_B}$ are the luminosity factors of the primaries, and have the values 1·00, 2·858 and 0·169.

λ.	Trichromatic coefficients.			${}_u\mathrm{L}_\lambda$.
	${}_u a_\lambda$.	${}_u b_\lambda$.	${}_u c_\lambda$.	
0·38	0·0683	— 0·0819	1·0136	0·0055
0·385	0·0683	— 0·0819	1·0136	0·0055
0·39	0·0683	— 0·0819	1·0136	0·0055
0·395	0·0683	— 0·0819	1·0136	0·0055
0·40	0·0683	— 0·0819	1·0136	0·0055
0·405	0·0670	— 0·0808	1·0138	0·0074
0·41	0·0655	— 0·0795	1·0140	0·0097
0·415	0·0638	— 0·0780	1·0142	0·0123
0·42	0·0613	— 0·0754	1·0141	0·0172
0·425	0·0590	— 0·0725	1·0135	0·0231
0·43	0·0554	— 0·0688	1·0134	0·0300
0·435	0·0502	— 0·0646	1·0144	0·0370
0·44	0·0443	— 0·0596	1·0153	0·0456
0·445	0·0365	— 0·0528	1·0163	0·0574
0·45	0·0262	— 0·0448	1·0186	0·0703
0·455	0·0130	— 0·0327	1·0197	0·0919
0·46	— 0·0030	— 0·0175	1·0205	0·1194
0·465	— 0·0255	+ 0·0056	1·0199	0·1629
0·47	— 0·0554	0·0420	1·0134	0·2359
0·475	— 0·0980	0·1010	0·9970	0·3592
0·48	— 0·1531	0·1843	0·9688	0·5374
0·485	— 0·2290	0·3125	0·9165	0·8190
0·49	— 0·3178	0·4839	0·8339	1·2061
0·495	— 0·4120	0·6964	0·7156	1·6992
0·50	— 0·5010	0·9247	0·5763	2·2392
0·505	— 0·5550	1·1290	0·4260	2·7437
0·51	— 0·5660	1·2870	0·2790	3·1594
0·515	— 0·5250	1·3670	0·1580	3·4086

TABLE II (continued).

λ.	Trichromatic coefficients.			$_uL_λ$.
	$_ua_λ$.	$_ub_λ$.	$_uc_λ$.	
0·52	− 0·4440	1·3620	0·0820	3·4625
0·525	− 0·3430	1·3020	0·0410	3·3850
0·53	− 0·2390	1·2230	0·0160	3·2590
0·535	− 0·1417	1·1410	0·0007	3·1194
0·54	− 0·0500	1·0590	− 0·0090	2·9751
0·545	+ 0·0404	0·9740	− 0·0144	2·8217
0·55	0·1279	0·8890	− 0·0169	2·6658
0·555	0·2143	0·8030	− 0·0173	2·5064
0·56	0·2977	0·7190	− 0·0167	2·3498
0·565	0·3796	0·6360	− 0·0156	2·1946
0·57	0·4600	0·5540	− 0·0140	2·0410
0·575	0·5380	0·4740	− 0·0120	1·8907
0·58	0·6120	0·3980	− 0·0100	1·7478
0·585	0·6815	0·3270	− 0·0085	1·6146
0·59	0·7427	0·2640	− 0·0067	1·4961
0·595	0·7980	0·2070	− 0·0050	1·3888
0·60	0·8465	0·1570	− 0·0035	1·2946
0·605	0·8876	0·1150	− 0·0026	1·2158
0·61	0·9238	0·0780	− 0·0018	1·1464
0·615	0·9523	0·0490	− 0·0013	1·0921
0·62	0·9748	0·0260	− 0·0008	1·0490
0·625	0·9925	0·0078	− 0·0003	1·0147
0·63	1·0068	− 0·0068	0·0000	0·9874
0·635	1·0188	− 0·0188	0·0000	0·9651
0·64	1·0290	− 0·0290	0·0000	0·9461
0·645	1·0370	− 0·0370	0·0000	0·9313
0·65	1·0430	− 0·0430	0·0000	0·9201
0·655	1·0480	− 0·0480	0·0000	0·9108
0·66	1·0509	− 0·0509	0·0000	0·9054
0·665	1·0532	− 0·0532	0·0000	0·9012
0·67	1·0550	− 0·0550	0·0000	0·8978
0·675	1·0565	− 0·0565	0·0000	0·8950
0·68	1·0580	− 0·0580	0·0000	0·8922
0·685	1·0590	− 0·0590	0·0000	0·8904
0·69	1·0599	− 0·0599	0·0000	0·8887
0·695	1·0603	− 0·0603	0·0000	0·8880
0·70	1·0604	− 0·0604	0·0000	0·8878

TABLE III.—Mean Spectral Measurements for Seven Observers tested at the National Physical Laboratory.

λ = wave-length (in μ).

${}_{u}a_{\lambda}$, etc. = coefficients in the expression ${}_{u}a_{\lambda}\mathrm{R} + {}_{u}b_{\lambda}\mathrm{G} + {}_{u}c_{\lambda}\mathrm{B}$, in which ${}_{u}a_{\lambda} + {}_{u}b_{\lambda} + {}_{u}c_{\lambda} = 1$. This represents the colorimetric quality of a monochromatic stimulus of wave-length λ in terms of R, G and B, which denote the primaries of the N.P.L. standard reference system, namely, monochromatic stimuli of wave-lengths $0{\cdot}700\,\mu$, $0{\cdot}5461\,\mu$, and $0{\cdot}5358\,\mu$.

${}_{w}a_{\lambda}$, etc. = ordinates of the distribution curves of the primaries for the spectrum of the N.P.L. standard white light (Table I). The numerical scale is such that ${}_{w}a_{\lambda}\mathrm{L_R} + {}_{w}b_{\lambda}\mathrm{L_G} + {}_{w}c_{\lambda}\mathrm{L_B} = \mathrm{E}_{\lambda}\mathrm{V}_{\lambda}$, the ordinate of the luminosity curve of the white light, where $\mathrm{L_R}$, $\mathrm{L_G}$ and $\mathrm{L_B}$ are the luminosity factors of the primaries and have the values 1·00, 4·410 and 0·052.

${}_{e}a_{\lambda}$, etc. = ordinate of the distribution curves of the primaries for an equal-energy spectrum in which E has the arbitrary value 100 at all wave-lengths.

${}_{u}\mathrm{L}_{\lambda}$ = luminosity factor of one "trichromatic unit" of monochromatic stimulus of wave-length λ, $= {}_{u}a_{\lambda}\mathrm{L_R} + {}_{u}b_{\lambda}\mathrm{L_G} + {}_{u}c_{\lambda}\mathrm{L_B}$.

NOTE.—In the same way as the ordinates of the white light distribution curves, when weighted by the luminosity factors of the primaries, summate to give the luminosity curve of the white light, those of the equal-energy distribution curves, when similarly weighted, summate to give the luminosity curve of the equal-energy spectrum, *i.e.*, the standard visibility curve.

λ.	Trichromatic Coefficients.			${}_{u}\mathrm{L}_{\lambda}$	Ordinates of Spectral Distribution Curves.					
					N.P.L. White Light.			Equal-energy Spectrum.		
	${}_{u}a_{\lambda}$.	${}_{u}b_{\lambda}$.	${}_{u}c_{\lambda}$.		${}_{w}a_{\lambda}$.	${}_{w}b_{\lambda}$.	${}_{w}c_{\lambda}$.	${}_{e}a_{\lambda}$.	${}_{e}b_{\lambda}$.	${}_{e}c_{\lambda}$.
0·38	0·0228	—0·0152	0·9924	0·0074	0·0000	0·000	0·000	0·0000	0·0000	0·0000
0·385	0·0228	—0·0152	0·9924	0·0074	0·0062	—0·004	0·268	0·0194	—0·0125	0·8375
0·39	0·0228	—0·0152	0·9924	0·0074	0·0154	—0·010	0·671	0·0434	—0·0282	1·890
0·395	0·0228	—0·0152	0·9924	0·0074	0·031	—0·021	1·341	0·0797	—0·0538	3·438
0·40	0·0228	—0·0152	0·9924	0·0074	0·052	—0·035	2·280	0·1204	—0·0810	5·278
0·405	0·0212	—0·0142	0·9930	0·0102	0·075	—0·050	3·504	0·158	—0·105	7·392
0·41	0·0196	—0·0133	0·9937	0·0126	0·096	—0·065	4·890	0·187	—0·127	9·485
0·415	0·0174	—0·0119	0·9945	0·0166	0·126	—0·086	7·186	0·226	—0·154	12·901
0·42	0·0144	—0·0097	0·9953	0·0234	0·148	—0·100	10·251	0·246	—0·166	17·042
0·425	0·0117	—0·0073	0·9956	0·0313	0·177	—0·110	15·046	0·277	—0·172	23·509
0·43	0·0073	—0·0041	0·9968	0·0410	0·139	—0·078	18·988	0·207	—0·116	28·214
0·435	0·0011	—0·0006	0·9995	0·0505	0·025	—0·014	22·979	0·036	—0·020	32·780
0·44	—0·0060	+0·0037	1·0023	0·0624	—0·161	+0·099	26·825	—0·222	+0·136	36·949
0·445	—0·0155	0·0095	1·0060	0·0787	—0·448	0·275	29·080	—0·602	0·370	39·086

TABLE III (continued).

λ.	Trichromatic Coefficients.			$_uL_\lambda$.	Ordinates of Spectral Distribution Curves. N.P.L. White Light.			Ordinates of Spectral Distribution Curves. Equal-energy Spectrum.		
	$_ua_\lambda$.	$_ub_\lambda$.	$_uc_\lambda$.		$_wa_\lambda$.	$_wb_\lambda$.	$_wc_\lambda$.	$_ea_\lambda$.	$_eb_\lambda$.	$_ec_\lambda$.
0·45	—0·0280	0·0163	1·0117	0·0965	—0·837	0·487	30·246	—1·103	0·642	39·850
0·455	—0·0443	0·0267	1·0176	0·1263	—1·307	0·787	30·012	—1·695	1·021	38·926
0·46	—0·0644	0·0400	1·0244	0·1653	—1·827	1·135	29·065	—2·336	1·451	37·168
0·465	—0·0925	0·0604	1·0321	0·2276	—2·386	1·558	26·619	—3·013	1·967	33·610
0·47	—0·1312	0·0930	1·0382	0·3329	—2·873	2·036	22·735	—3·587	2·542	28·383
0·475	—0·1881	0·1475	1·0406	0·5165	—3·325	2·607	18·395	—4·115	3·226	22·766
0·48	—0·2651	0·2274	1·0377	0·7919	—3·780	3·242	14·794	—4·655	3·993	18·219
0·485	—0·3788	0·3577	1·0211	1·2513	—4·211	3·976	11·351	—5·167	4·879	13·928
0·49	—0·5254	0·5465	0·9789	1·9355	—4·609	4·794	8·588	—5·645	5·871	10·518
0·495	—0·7019	0·8049	0·8970	2·8947	—5·082	5·828	6·495	—6·232	7·147	7·964
0·50	—0·8948	1·1165	0·7783	4·0697	—5·743	7·166	4·995	—7·099	8·858	6·174
0·505	—1·0395	1·4224	0·6171	5·2656	—6·396	8·752	3·797	—8·020	10·974	4·761
0·51	—1·0985	1·6678	0·4307	6·2789	—6·958	10·564	2·728	—8·796	13·355	3·449
0·515	—1·0326	1·7733	0·2593	6·8011	—7·311	12·555	1·836	—9·208	15·812	2·312
0·52	—0·8684	1·7228	0·1456	6·7367	—7·421	14·723	1·244	—9·150	18·154	1·534
0·525	—0·6669	1·5838	0·0831	6·3220	—7·012	16·652	0·874	—8·358	19·847	1·042
0·53	—0·4739	1·4283	0·0456	5·8275	—6·080	18·326	0·585	—7·009	21·125	0·674
0·535	—0·3070	1·2838	0·0232	5·3562	—4·685	19·592	0·354	—5·229	21·866	0·395
0·54	—0·1613	1·1520	0·0093	4·9192	—2·888	20·629	0·167	—3·127	22·338	0·181
0·545	—0·0277	1·0264	0·0013	4·4984	—0·574	21·254	0·027	—0·605	22·420	0·028
0·55	+0·0928	0·9102	—0·0030	4·1066	+2·183	21·415	—0·071	+2·248	22·055	—0·073
0·555	0·2040	0·8007	—0·0047	3·7348	5·397	21·184	—0·124	5·457	21·420	—0·125
0·56	0·3047	0·7006	—0·0053	3·3944	8·932	20·537	—0·155	8·932	20·537	—0·155
0·565	0·3976	0·6076	—0·0052	3·0768	12·714	19·430	—0·166	12·676	19·372	—0·166
0·57	0·4837	0·5210	—0·0047	2·7811	16·575	17·853	—0·161	16·558	17·835	—0·161
0·575	0·5627	0·4412	—0·0039	2·5082	20·261	15·886	—0·140	20·404	15·998	—0·141
0·58	0·6338	0·3691	—0·0029	2·2614	23·865	13·898	—0·109	24·377	14·196	—0·111
0·585	0·6976	0·3049	—0·0025	2·0421	27·086	11·839	—0·097	28·171	12·313	—0·101
0·59	0·7513	0·2504	—0·0017	1·8555	29·015	9·670	—0·066	30·639	10·211	—0·070
0·595	0·7981	0·2028	—0·0009	1·6924	30·558	7·765	—0·034	32·647	8·296	—0·036
0·60	0·8352	0·1650	—0·0002	1·5629	31·074	6·139	—0·007	33·721	6·662	—0·008
0·605	0·8706	0·1294	0·0000	1·4412	30·966	4·602	0·000	34·179	5·079	0·000
0·61	0·8990	0·1010	0·0000	1·3444	30·091	3·381	0·000	33·640	3·780	0·000
0·615	0·9208	0·0792	0·0000	1·2701	28·478	2·449	0·000	32·106	2·761	0·000
0·62	0·9379	0·0621	0·0000	1·2118	25·983	1·720	0·000	29·509	1·953	0·000
0·625	0·9512	0·0488	0·0000	1·1664	22·908	1·175	0·000	26·151	1·341	0·000
0·63	0·9618	0·0382	0·0000	1·1303	19·707	0·783	0·000	22·548	0·896	0·000
0·635	0·9705	0·0295	0·0000	1·1006	16·710	0·508	0·000	19·119	0·581	0·000
0·64	0·9778	0·0222	0·0000	1·0757	13·943	0·317	0·000	15·908	0·362	0·000
0·645	0·9835	0·0165	0·0000	1·0563	11·359	0·191	0·000	12·930	0·217	0·000

TABLE III (continued).

λ.	Trichromatic Coefficients.			$_uL_λ$.	Ordinates of Spectral Distribution Curves. N.P.L. White Light.			Ordinates of Spectral Distribution Curves. Equal-energy Spectrum.		
	$_ua_λ$.	$_ub_λ$.	$_uc_λ$.		$_wa_λ$.	$_wb_λ$.	$_wc_λ$.	$_ea_λ$.	$_eb_λ$.	$_ec_λ$.
0·65	0·9877	0·0123	0·0000	1·0419	8·921	0·111	0·000	10·143	0·126	0·000
0·655	0·9913	0·0087	0·0000	1·0297	6·970	0·061	0·000	7·925	0·069	0·000
0·66	0·9933	0·0067	0·0000	1·0228	5·206	0·035	0·000	5·923	0·040	0·000
0·665	0·9949	0·0051	0·0000	1·0174	3·765	0·019	0·000	4·293	0·022	0·000
0·67	0·9962	0·0038	0·0000	1·0130	2·753	0·011	0·000	3·146	0·013	0·000
0·675	0·9973	0·0027	0·0000	1·0092	2·006	0·005	0·000	2·299	0·006	0·000
0·68	0·9983	0·0017	0·0000	1·0058	1·469	0·003	0·000	1·689	0·003	0·000
0·685	0·9990	0·0010	0·0000	1·0034	1·036	0·001	0·000	1·196	0·001	0·000
0·69	0·9996	0·0004	0·0000	1·0014	0·709	0·000	0·000	0·822	0·000	0·000
0·695	0·9999	0·0001	0·0000	1·0003	0·503	0·000	0·000	0·585	0·000	0·000
0·70	1·0000	0·0000	0·0000	1·0000	0·351	0·000	0·000	0·410	0·000	0·000
0·705	1·0000	0·0000	0·0000	1·0000	0·251	0·000	0·000	0·295	0·000	0·000
0·71	1·0000	0·0000	0·0000	1·0000	0·178	0·000	0·000	0·210	0·000	0·000
0·715	1·0000	0·0000	0·0000	1·0000	0·121	0·000	0·000	0·144	0·000	0·000
0·72	1·0000	0·0000	0·0000	1·0000	0·087	0·000	0·000	0·105	0·000	0·000
0·725	1·0000	0·0000	0·0000	1·0000	0·063	0·000	0·000	0·076	0·000	0·000
0·73	1·0000	0·0000	0·0000	1·0000	0·043	0·000	0·000	0·052	0·000	0·000
0·735	1·0000	0·0000	0·0000	1·0000	0·029	0·000	0·000	0·036	0·000	0·000
0·74	1·0000	0·0000	0·0000	1·0000	0·020	0·000	0·000	0·025	0·000	0·000
0·745	1·0000	0·0000	0·0000	1·0000	0·013	0·000	0·000	0·016	0·000	0·000
0·750	1·0000	0·0000	0·0000	1·0000	0·009	0·000	0·000	0·012	0·000	0·000
0·755	1·0000	0·0000	0·0000	1·0000	0·007	0·000	0·000	0·009	0·000	0·000
0·760	1·0000	0·0000	0·0000	1·0000	0·005	0·000	0·000	0·006	0·000	0·000

TABLE IV.—Proposed Spectral Data for "Normal" Eye, based on Mean Colorimetric Results of GUILD and WRIGHT.

λ = wave-length (in μ).

$_ua_\lambda$, etc. = coefficients in the expression $_ua_\lambda R + {}_ub_\lambda G + {}_uc_\lambda B$, in which $_ua_\lambda + {}_ub_\lambda + {}_uc_\lambda = 1$. This represents the colorimetric quality of a monochromatic stimulus of wave-length λ in terms of R, G and B, which denote the primaries of the N.P.L. standard reference system, namely, monochromatic stimuli of wave-lengths $0 \cdot 700\mu$, $0 \cdot 5461\mu$ and $0 \cdot 4358\mu$.

$_wa_\lambda$, etc. = ordinates of the distribution curves of the primaries for the spectrum of the N.P.L. standard white light (Table I). The numerical scale is such that $_wa_\lambda L_R + {}_wb_\lambda L_G + {}_wc_\lambda L_B = E_\lambda V_\lambda$, the ordinate of the luminosity curve of the white light, where L_R, L_G and L_B are the luminosity factors of the primaries, and have the values 1·00, 4·390 and 0·048.

$_ea_\lambda$, etc. = ordinates of the distribution curves of the primaries for an equal energy spectrum in which E has the arbitrary value 100 at all wave-lengths.

$_uL_\lambda$ = luminosity factor of one "trichromatic unit" of monochromatic stimulus of wave-length λ, $= {}_ua_\lambda L_R + {}_ub_\lambda L_G + {}_uc_\lambda L_B$.

NOTE.—In the same way as the ordinates of the white light distribution curves, when weighted by the luminosity factors of the primaries, summate to give the luminosity curve of the white light, those of the equal-energy distribution curves, when similarly weighted, summate to give the luminosity curve of the equal energy spectrum, *i.e.*, the standard visibility curve.

λ.	Trichromatic Coefficients.			$_uL_\lambda$.	Ordinates of Spectral Distribution Curves.					
					N.P.L. White Light.			Equal-energy Spectrum.		
	$_ua_\lambda$.	$_ub_\lambda$.	$_uc_\lambda$.		$_wa_\lambda$.	$_wb_\lambda$.	$_wc_\lambda$.	$_ea_\lambda$.	$_eb_\lambda$.	$_ec_\lambda$.
0·38	0·0204	—0·0106	0·9902	0·0214	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000
0·385	0·0204	—0·0106	0·9902	0·0214	0·00084	—0·00044	0·041	0·0026	—0·0014	0·128
0·39	0·0204	—0·0106	0·9902	0·0214	0·0031	—0·0016	0·150	0·0087	—0·0045	0·423
0·395	0·0204	—0·0106	0·9902	0·0214	0·0075	—0·0039	0·364	0·0192	—0·0100	0·933
0·40	0·0204	—0·0106	0·9902	0·0214	0·0165	—0·0086	0·800	0·0381	—0·0198	1·851
0·405	0·0191	—0·0101	0·9910	0·0223	0·0307	—0·0163	1·595	0·0649	—0·0343	3·366
0·41	0·0177	—0·0096	0·9919	0·0232	0·047	—0·0256	2·648	0·092	—0·0497	5·136
0·415	0·0162	—0·0089	0·9927	0·0248	0·078	—0·043	4·798	0·141	—0·077	8·611
0·42	0·0137	—0·0074	0·9937	0·0289	0·114	—0·062	8·271	0·190	—0·102	13·750
0·425	0·0114	—0·0060	0·9946	0·0328	0·164	—0·087	14·346	0·257	—0·135	22·422
0·43	0·0072	—0·0037	0·9965	0·0388	0·145	—0·074	20·061	0·215	—0·111	29·804
0·435	0·0011	—0·0005	0·9994	0·0469	0·027	—0·012	24·760	0·039	—0·018	35·310
0·44	—0·0070	+0·0039	1·0031	0·0583	—0·201	+0·112	28·737	—0·276	+0·154	39·599
0·445	—0·0178	0·0093	1·0085	0·0714	—0·567	0·296	32·125	—0·762	0·398	43·161

TABLE IV (continued).

λ.	Trichromatic Coefficients.			${}_uL_\lambda$.	Ordinates of Spectral Distribution Curves. N.P.L. White Light.			Ordinates of Spectral Distribution Curves. Equal-energy Spectrum.		
	${}_ua_\lambda$.	${}_ub_\lambda$.	${}_uc_\lambda$.		${}_wa_\lambda$.	${}_wb_\lambda$.	${}_wc_\lambda$.	${}_ea_\lambda$.	${}_eb_\lambda$.	${}_ec_\lambda$.
0·45	—0·0315	0·0161	1·0154	0·0879	—1·033	0·528	33·298	—1·362	0·696	43·889
0·455	—0·0487	0·0264	1·0223	0·1163	—1·560	0·846	32·751	—2·024	1·097	42·479
0·46	—0·0697	0·0405	1·0292	0·1575	—2·076	1·207	30·661	—2·656	1·543	39·223
0·465	—0·0983	0·0632	1·0351	0·2288	—2·521	1·621	26·547	—3·182	2·046	33·511
0·47	—0·1376	0·0970	1·0406	0·3382	—2·967	2·091	22·435	—3·703	2·610	28·002
0·475	—0·1990	0·1520	1·0470	0·5185	—3·505	2·677	18·440	—4·335	3·312	22·810
0·48	—0·2846	0·2367	1·0479	0·8048	—3·992	3·320	14·698	—4·916	4·089	18·101
0·485	—0·4019	0·3689	1·0330	1·267	—4·411	4·049	11·338	—5·411	4·966	13·907
0·49	—0·5527	0·5613	0·9914	1·959	—4·792	4·866	8·595	—5·869	5·960	10·527
0·495	—0·7480	0·8440	0·9040	3·001	—5·225	5·896	6·315	—6·406	7·229	7·743
0·50	—0·9494	1·1727	0·7767	4·236	—5·855	7·232	4·790	—7·240	8·942	5·923
0·505	—1·0800	1·4750	0·6050	5·424	—6·451	8·810	3·614	—8·088	11·045	4·531
0·51	—1·1203	1·6964	0·4239	6·347	—7·018	10·628	2·656	—8·876	13·441	3·359
0·515	—1·0311	1·7647	0·2664	6·729	—7·378	12·627	1·906	—9·292	15·904	2·401
0·52	—0·8637	1·7114	0·1523	6·657	—7·471	14·804	1·317	—9·214	18·258	1·625
0·525	—0·6739	1·5849	0·0890	6·288	—7·124	16·754	0·941	—8·492	19·972	1·122
0·53	—0·4879	1·4406	0·0473	5·839	—6·248	18·448	0·606	—7·203	21·268	0·698
0·535	—0·3165	1·2944	0·0221	5·367	—4·821	19·715	0·337	—5·381	22·008	0·376
0·54	—0·1617	1·1530	0·0087	4·900	—2·907	20·727	0·156	—3·148	22·446	0·169
0·545	—0·0279	1·0267	0·0012	4·479	—0·580	21·351	0·025	—0·612	22·525	0·026
0·55	+0·0859	0·9166	—0·0025	4·110	+2·020	21·551	—0·059	+2·080	22·195	—0·061
0·555	0·1990	0·8064	—0·0054	3·739	5·259	21·312	—0·143	5·318	21·551	—0·144
0·56	0·3032	0·7029	—0·0061	3·389	8·904	20·639	—0·180	8·904	20·639	—0·180
0·565	0·3988	0·6068	—0·0056	3·062	12·813	19·496	—0·180	12·771	19·432	—0·179
0·57	0·4864	0·5185	—0·0049	2·762	16·782	17·889	—0·169	16·767	17·873	—0·169
0·575	0·5664	0·4376	—0·0040	2·487	20·565	15·888	—0·145	20·708	15·999	—0·146
0·58	0·6376	0·3655	—0·0031	2·242	24·221	13·882	—0·117	24·748	14·184	—0·120
0·585	0·7003	0·3020	—0·0023	2·026	27·407	11·819	—0·090	28·507	12·294	—0·094
0·59	0·7572	0·2442	—0·0014	1·829	29·674	9·570	—0·055	31·344	10·109	—0·058
0·595	0·8031	0·1979	—0·0010	1·672	31·126	7·670	—0·039	33·258	8·195	—0·041
0·60	0·8406	0·1600	—0·0006	1·543	31·679	6·030	—0·023	34·374	6·543	—0·025
0·605	0·8738	0·1267	—0·0005	1·430	31·324	4·542	—0·018	34·574	5·013	—0·020
0·61	0·9010	0·0995	—0·0005	1·338	30·310	3·347	—0·017	33·881	3·742	—0·019
0·615	0·9219	0·0786	—0·0005	1·267	28·586	2·437	—0·016	32·224	2·747	—0·017
0·62	0·9385	0·0620	—0·0005	1·211	26·020	1·719	—0·014	29·552	1·952	—0·016
0·625	0·9526	0·0479	—0·0005	1·163	23·008	1·157	—0·012	26·256	1·320	—0·014
0·63	0·9639	0·0366	—0·0005	1·125	19·852	0·754	—0·010	22·714	0·862	—0·012
0·635	0·9732	0·0273	—0·0005	1·093	16·876	0·473	—0·009	19·309	0·542	—0·010
0·64	0·9799	0·0206	—0·0005	1·070	14·047	0·295	—0·007	16·024	0·337	—0·008
0·645	0·9853	0·0152	—0·0005	1·052	11·428	0·176	—0·005	13·006	0·201	—0·007

TABLE IV (continued).

λ.	Trichromatic Coefficients.			$_uL_\lambda$.	Ordinates of Spectral Distribution Curves.					
					N.P.L. White Light.			Equal-energy Spectrum.		
	$_ua_\lambda$.	$_ub_\lambda$.	$_uc$.		$_wa_\lambda$.	$_wb_\lambda$.	$_wc_\lambda$.	$_ea_\lambda$.	$_eb_\lambda$.	$_ec_\lambda$.
0·65	0·9889	0·0111	0·0000	1·038	8·969	0·101	0·000	10·200	0·114	0·000
0·655	0·9917	0·0083	0·0000	1·028	6·989	0·059	0·000	7·949	0·067	0·000
0·66	0·9937	0·0063	0·0000	1·021	5·215	0·033	0·000	5·934	0·038	0·000
0·665	0·9950	0·0050	0·0000	1·017	3·765	0·019	0·000	4·292	0·022	0·000
0·67	0·9961	0·0039	0·0000	1·013	2·754	0·011	0·000	3·147	0·012	0·000
0·675	0·9971	0·0029	0·0000	1·010	2·005	0·006	0·000	2·297	0·007	0·000
0·68	0·9976	0·0024	0·0000	1·008	1·470	0·004	0·000	1·691	0·004	0·000
0·685	0·9985	0·0015	0·0000	1·005	1·037	0·002	0·000	1·196	0·002	0·000
0·69	0·9993	0·0007	0·0000	1·002	0·706	0·001	0·000	0·818	0·001	0·000
0·695	0·9997	0·0003	0·0000	1·001	0·503	0·000	0·000	0·584	0·000	0·000
0·70	1·0000	0·0000	0·0000	1·000	0·351	0·000	0·000	0·410	0·000	0·000
0·705	1·0000	0·0000	0·0000	1·000	0·251	0·000	0·000	0·295	0·000	0·000
0·71	1·0000	0·0000	0·0000	1·000	0·178	0·000	0·000	0·210	0·000	0·000
0·715	1·0000	0·0000	0·0000	1·000	0·121	0·000	0·000	0·144	0·000	0·000
0·72	1·0000	0·0000	0·0000	1·000	0·087	0·000	0·000	0·105	0·000	0·000
0·725	1·0000	0·0000	0·0000	1·000	0·063	0·000	0·000	0·077	0·000	0·000
0·73	1·0000	0·0000	0·0000	1·000	0·043	0·000	0·000	0·053	0·000	0·000
0·735	1·0000	0·0000	0·0000	1·000	0·029	0·000	0·000	0·036	0·000	0·000
0·74	1·0000	0·0000	0·0000	1·000	0·020	0·000	0·000	0·025	0·000	0·000
0·745	1·0000	0·0000	0·0000	1·000	0·013	0·000	0·000	0·017	0·000	0·000
0·75	1·0000	0·0000	0·0000	1·000	0·009	0·000	0·000	0·012	0·000	0·000
0·755	1·0000	0·0000	0·0000	1·000	0·007	0·000	0·000	0·009	0·000	0·000
0·76	1·0000	0·0000	0·0000	1·000	0·005	0·000	0·000	0·007	0·000	0·000
0·765	1·0000	0·0000	0·0000	1·000	0·003	0·000	0·000	0·004	0·000	0·000
0·77	1·0000	0·0000	0·0000	1·000	0·002	0·000	0·000	0·003	0·000	0·000
0·775	1·0000	0·0000	0·0000	1·000	0·001	0·000	0·000	0·001	0·000	0·000
0·78	1·0000	0·0000	0·0000	1·000	0·000	0·000	0·000	0·000	0·000	0·000